Chapter 2: Matrix Algebra

Section 2.1 – Matrix Operations

Terminology: Let A be an $m \times n$ matrix.

1. We denote the entry of A in row i and column j by

2. The **diagonal entries** of A are

3. A is the **zero matrix**

4. A is a **diagonal matrix** if A

Examples:

1. $A = \begin{bmatrix} 1 & 3 & -25 \\ 2 & 10 & 1 \\ -1 & 8 & 2 \\ 0 & 20 & 6 \end{bmatrix}$

2. $\begin{bmatrix} -5 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}$

3.

Definitions: Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be $m \times n$ matrices.

1. A **equals** B if

2. The **sum** $A + B$ is the $m \times n$ matrix whose

3. If r is a scalar, then the **scalar multiple** rA is the $m \times n$ matrix whose

Note: $-A = (-1)A$ and

Examples: Let $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 3 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 8 \\ -1 & 1 \\ 2 & 0 \end{bmatrix}$.

Theorem: Let A, B and C be $m \times n$ matrices and let r, s be scalars. Then

1. $A + B =$

2. $(A + B) + C =$

3. $A + 0 =$

4. $r(A + B) =$

5. $(r + s)A =$

6. $r(sA) =$

Matrix Multiplication:

Goal: To find the standard matrix of the composition of two linear transformations.

We know that B is an $n \times p$ matrix, A is an $m \times n$ matrix. Let $\mathbf{x}$ be in $\mathbb{R}^p$ and denote B by $B = [\mathbf{b_1} \quad \cdots \quad \mathbf{b_p}]$. Then

Definition: If A is an $m \times n$ matrix and B is an $n \times p$ matrix with columns $\mathbf{b_1}, \ldots, \mathbf{b_p}$, then

Note:

Examples:

1. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 & 1 \\ 1 & -2 & 3 \end{bmatrix}$.

2. $\begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & -4 \\ 6 & -8 & -7 \\ -3 & 0 & 9 \end{bmatrix} \begin{bmatrix} 4 & -6 \\ 7 & 1 \\ 3 & 3 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix}$

Theorem: Let A be an $m \times n$ matrix and B and C be matrices of appropriate sizes. Then

1. $A(BC) =$

2. $A(B + C) =$

3. $(B + C)A =$

4. $r(AB) = (rA)B = A(rB)$

5. $I_m A =$