Chapter 2: Matrix Algebra

Section 2.1 – Matrix Operations (Continued)

Food For Thought Examples:

1. If $A = \begin{bmatrix} 5 & 1 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$, then

2. Let $A = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix}$, and $C = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$. Note that

3. If $A = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$, then

Note: If A is an $n \times n$ matrix, then

Definition: Let A be an $m \times n$ matrix. The **transpose** of A, denoted A^T, is the $n \times m$ matrix whose columns are formed from the

Examples:

1. $A = \begin{bmatrix} -4 & -1 \\ 2 & 2 \\ 0 & 3 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & -2 & 1 & 7 \\ 0 & 5 & 3 & 10 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

4. $A = \begin{bmatrix} 5 & 1 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

Theorem: Let A and B be matrices of appropriate sizes. Then

(a) $(A^T)^T =$

(b) $(A + B)^T =$

(c) For any scalar r, $(rA)^T =$

(d) $(AB)^T =$

Section 2.2 – The Inverse of a Matrix

Goal: The inverse of 3 is $3^{-1} = 1/3$ since

$$3^{-1} \cdot 3 = 1 \quad \text{and} \quad 3 \cdot 3^{-1} = 1.$$

We want to mimic this for matrices!

Definition: An $n \times n$ matrix A is said to be **invertible** if there exists an $n \times n$ matrix C such that

Fact: C is uniquely determined by A! To see this, let B be another inverse of A. Then

Definition:

1. A matrix which is not invertible is called

2. An invertible matrix is called

Useful Facts for Invertible Matrices:

Theorem: Let A and B be $n \times n$ invertible matrices.

1. For each $\mathbf{b}$ in $\mathbb{R}^n$, the equation $A\mathbf{x} = \mathbf{b}$ has the unique solution
$$\mathbf{x} = A^{-1}\mathbf{b}.$$
2. A^{-1} is invertible and
$$(A^{-1})^{-1} = A.$$
3. AB is invertible and
$$(AB)^{-1} = B^{-1}A^{-1}.$$
4. A^T is invertible and
$$(A^T)^{-1} = (A^{-1})^T.$$