# Chapter 1: Linear Equations in Linear Algebra

## Section 1.1 – Systems of Linear Equations (Continued)

**Observe:** When we solve a linear system we replace it with an equivalent system which is easier to solve. To find the replacement we need to only keep track of coefficients in a systematic way. We can do **3** valid operations only to the augmented matrix when doing this:

**Elementary Row Operations of a Matrix:**

1. **Replacement:**

2. **Interchange:**

3. **Scaling:**

**Definition:** Two matrices are called **row equivalent** if there is a sequence of *elementary row operations* that transforms one matrix into the other.

**Fact:** If the augmented matrices of 2 linear systems are row equivalent, then the 2 systems

**Examples:**

1. Solve the linear system

$$\begin{aligned} x_1 - 3x_2 &= 5 \\ -x_1 + x_2 + 5x_3 &= 2 \\ x_2 + x_3 &= 0 \end{aligned}$$

2. Determine if the following system is consistent:

$$\begin{aligned} x_1 - 2x_2 - x_3 + 3x_4 &= 0 \\ -2x_1 + 4x_2 + 5x_3 - 5x_4 &= 3 \\ 3x_1 - 6x_2 - 6x_3 + 8x_4 &= 2 \end{aligned}$$

3. For what value(s) of $h$ does the linear system

$$\begin{aligned} 2x_1 - 3x_2 &= h \\ -6x_1 + 9x_2 &= 5 \end{aligned}$$

have a solution?

## Section 1.2 – Row Reduction and Echelon Forms

**Goal:** To study an algorithm used to solve linear systems of equations.

**Note:** When we solved linear systems in Section 1.1, we obtained "staircase" forms via elementary row operations. We give such forms special names.

**Definition:** A **leading entry** of a non-zero row in a matrix is the

**Definition:** A rectangular matrix is in **echelon form** (or **row echelon form**) if it satisfies:

1. All non-zero rows are above any

2. Each leading entry of a row is in a

3. All entries in a column below a leading entry are

A matrix in echelon form is in **reduced (row) echelon form** if it also satisfies:

4. The leading entry in each non-zero row is

5. Each leading 1 is the only non-zero entry

**Examples:**

1. $\begin{bmatrix} 5 & -2 & 2 & 0 \\ 0 & 6 & -1 & 8 \\ 0 & 0 & 0 & 5/2 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 0 & 0 & 16 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

**Theorem:** Each matrix is row equivalent to *one and only one* reduced (row) echelon matrix.