MATH 1230

September 7 Lecture 1

Please read the course outline carefully!

In particular, note that this course is $\boxed{\text{not}}$ the same as MATH 1500. It will be more difficult and will involve much more work, and is intended for students who will be continuing their studies in mathematically rich disciplines, e.g. math, physics.

What is calculus?

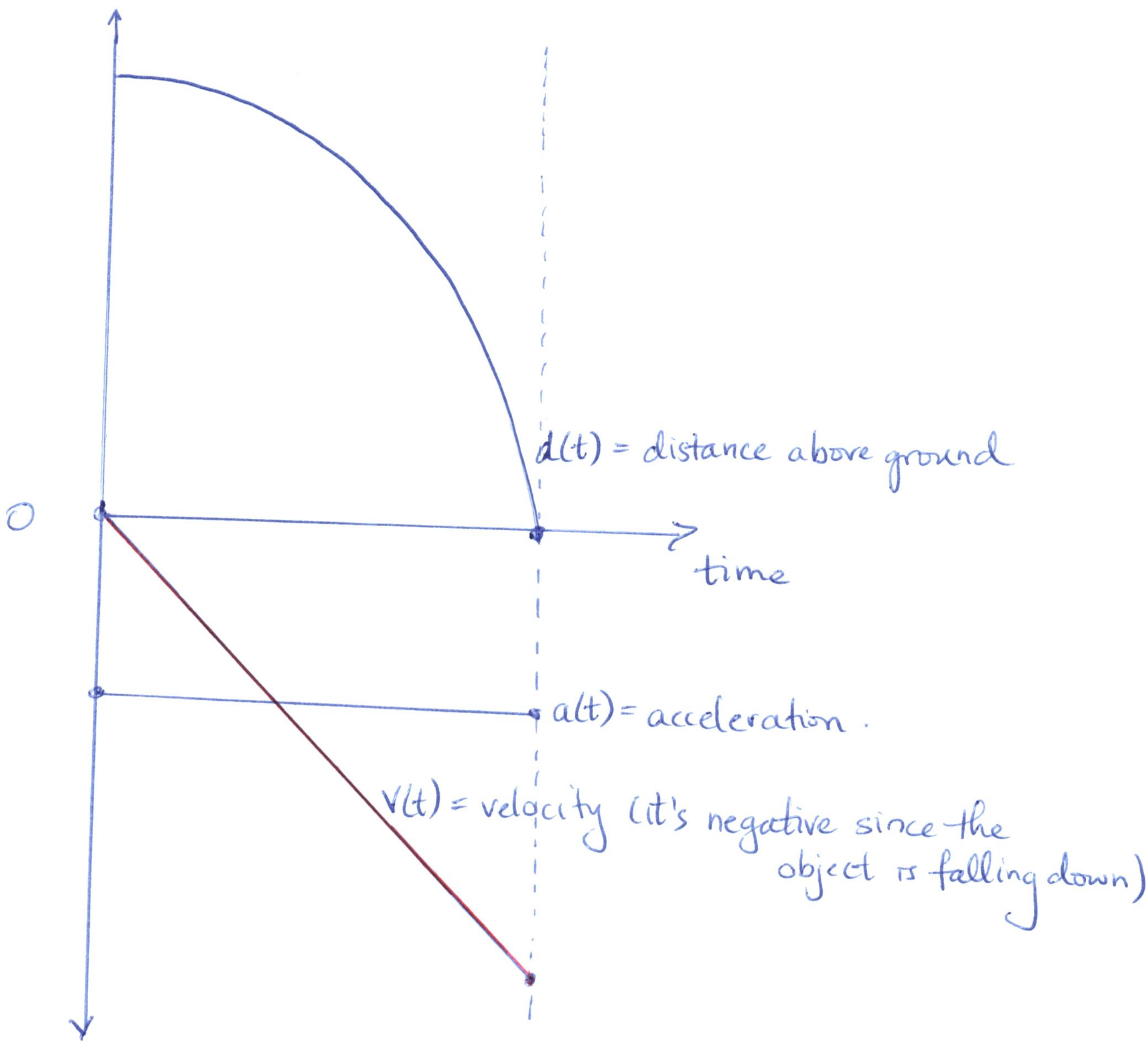
Calculus is a precise study of <u>rates of change</u>.

This is a natural perspective on many problems that arise in nature. For example, suppose we are studying a moving object, say a satellite falling to earth.

We could plot several different quantities:

1. Distance from the ground at time t
2. Velocity of the satellite at time t
3. Acceleration of the satellite at time t.

We might get something like this:



The units on the y-axis would be different for each graph:

m for distance

m/s for velocity

m/s^2 for acceleration.

<u>Note</u>: Each is the "rate of change" of the previous one.

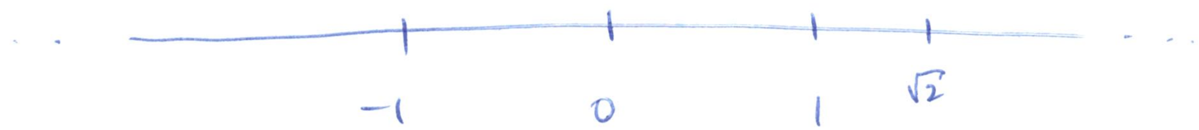
We will study the precise mathematical process that allows you to calculate acceleration from velocity, and velocity from distance.

in general, we will calculate the rate of change of a function.

Preliminaries: We will cover P1, P4, P5 and P7 carefully. Sections P2, P3, P6 are also required but you must study them *on your own* if you are uncertain about any of this background material.

The real numbers $\mathbb{R}$:

We draw them like this:



and use inequalities and/or intervals to indicate regions on the line.

Inequality	$-3 < x < 5$	$-1 \leq x < 1$	$-5 \leq x \leq 10$
Interval	$(-3, 5)$	$[-1, 1)$	$[-5, 10]$.

Note: Square brackets correspond to $\leq$ vs round brackets correspond to $<$. Therefore square brackets mean that we "include the endpoints".

Real numbers can be rational (a fraction) or irrational (*not* a fraction).

Rational numbers sometimes have a finite decimal expansion: $\frac{3}{4} = 0.75$.

Irrational numbers never have a finite decimal expansion. Therefore, in this class we never use decimal approximations of numbers.

Example: Prove that $\sqrt{2}$ is an irrational number.

Proof: What if it is rational?

Then $\sqrt{2} = \frac{p}{q}$ for some p and q that do not have a common divisor.

So $2 = \frac{p^2}{q^2} \Rightarrow 2q^2 = p^2$ (use $\Rightarrow$ to mean "implies").

Now $2q^2$ is an even number, so p^2 must be an even number, too. This means p must be even, so we can write $p = 2n$ for some n.

Then $2q^2 = p^2$ becomes

$$2q^2 = (2n)^2 = 4n^2 \Longrightarrow q^2 = 2n^2.$$

But now $2n^2$ is even, so q^2 is even, meaning q is even.

So, p and q are both even. This cannot be true, as we started with p and q having no common divisor.

Thus, $\sqrt{2}$ is irrational.

So, replacing $\sqrt{2}$ with any decimal approximation will always be wrong.

This course contains proofs. You will be expected to write proofs.

A "proof" is a careful, logical explanation of why something is true. For us, proofs will always be a series of mathematical statements, each one following logically from the previous one, that ends with the desired conclusion.

Proof writing can only be learned through practice and examples