Algebra Review

1. Express the following in the form x^r.

(a) $(\sqrt[5]{x})^6$ (b) $\sqrt[8]{x^3}$ (c) $\dfrac{1}{(\sqrt{x})^5}$ (d) $\dfrac{1}{\sqrt[3]{x^4}}$ (e) $\sqrt[4]{\sqrt[3]{x}}$ (f) $\sqrt{\dfrac{1}{\sqrt[5]{x}}}$

2. Express the following in the form x^r.

(a) $x^{\frac{5}{2}}x^3$ (b) $\dfrac{x^{\frac{6}{7}}}{x^4}$ (c) $(x^3)^{-\frac{1}{5}}$ (d) $x^{\frac{1}{5}}x^{-\frac{6}{5}}$ (e) $(x^{\frac{3}{5}})^{\frac{4}{5}}$ (f) $\dfrac{1}{x^{\frac{5}{2}}}$

(g) $\left(\dfrac{1}{x^3}\right)^{-\frac{2}{3}}$ (h) $\dfrac{1}{x\sqrt{x}}$ (i) $x^2(\sqrt[3]{x})$ (j) $\dfrac{x}{x^{\frac{2}{5}}}$ (k) $\dfrac{x^{\frac{1}{3}}}{x}$ (l) $\dfrac{1}{x^{-\frac{1}{4}}}$

3. Which of the following equations are true for all nonzero values of x and y?

(a) $x^5 + x^2 = x^7$ (b) $x^4x^7 = x^{11}$ (c) $(xy)^3 = x^3y^3$ (d) $(x^4)^3 = x^7$

(e) $(x+y)^4 = x^4 + y^4$ (f) $(x^3)^3 = x^9$ (g) $x^7 - x^3 = x^4$ (h) $\dfrac{x^8}{x^2} = x^4$

(i) $\dfrac{x^4}{y^4} = \left(\dfrac{x}{y}\right)^4$ (j) $\dfrac{x^9}{x^3} = x^6$ (k) $(x-y)^4 = \dfrac{x^4}{y^4}$ (l) $\dfrac{x^{10}}{y^2} = \left(\dfrac{x}{y}\right)^5$

4. Express the following as single polynomials.

(a) $(3x^3 - 4x^2 + 5x - 7) + (5x^3 + 4x^2 - 8x + 9)$ (b) $-(9x^3 - 4x^2 - 6x + 10)$

(c) $(2x^5 - x^3 + 4x^2 - x) - (2x^5 + x^4 - x^3 + 2x^2 - 7)$

(d) $5x^2 - 3x - \{3x^2 + 4x - [8x^2 - 2x - (7x^2 - 4)]\}$

(e) $(x^3 + 2x^2 - x - 4)(x^4 - 2x^3 + x^2 - 5)$

(f) $(x+1)(x+2)(x-3) - (x^2-1)(x-2)$ (g) $(x^2+2)^3 - (3x^3-4)^2$

5. Express the following as single polynomials.

(a) $(2x-3)^2$ (b) $(x+5)^3$ (c) $(2x-3)^3$ (d) $(x-2)^5$ (e) $(2x+1)^4$

6. Factor the following as completely as possible (using real numbers).

(a) $x^4 - 9x^2$ (b) $x^2 - 10$ (c) $21x^2 - 3x^4$

(d) $3x^5 - 24x^3$ (e) $7x^6 + 28x^4$ (f) $x^2 - 17x + 16$

(g) $-4x^4 + 20x^3 + 24x^2$ (h) $6x^2 - 17x + 7$ (i) $-18x^3 + 36x^2 - 16x$

(j) $x^4 - 81$ (k) $x^4 - 9$ (l) $x^4 + 5x^2 - 36$

(m) $3x^5 - 18x^3 + 15x$ (n) $-12x^6 - 46x^4 - 14x^2$ (o) $x^3 + 8$

(p) $8x^3 - 27$ (q) $x^3 + 3$ (r) $64x^8 + 27x^5$

(s) $4x^2 - 3x^5$

7. Reduce the following as much as possible.

(a) $\dfrac{x^2+6x+9}{x^2-9}$ (b) $\dfrac{x^2}{4x^2+7x}$ (c) $\dfrac{x^2+5x+6}{x^2+5x}$

(d) $\dfrac{x^3+8}{x^2-4}$ (e) $\dfrac{x^4+3x^3-10x^2}{4x^3-x^5}$

8. Express the following as rational terms with denominators of lowest possible degree.

(a) $\left(\dfrac{x^2-4}{x^2+x}\right)\left(\dfrac{x+1}{x+2}\right)$ (b) $\left(\dfrac{4x^3-3x^2-10x}{2x^2+12x+18}\right)\left(\dfrac{2x^2+4x-6}{3x^4-12x^3+12x^2}\right)$

(c) $\left(\dfrac{x^2+5x+4}{x^2-4}\right)\div\left(\dfrac{x^2-1}{x+2}\right)$ (d) $\left(\dfrac{4x^2-12x+9}{2x^2-5x+2}\right)\div\left(\dfrac{9-4x^2}{4x^2-1}\right)$

(e) $\dfrac{x}{x+3}+\dfrac{5x^2}{x^2-9}$ (f) $\dfrac{x}{x^2-2x+5}+\dfrac{3}{x-1}$

(g) $\dfrac{x+1}{x^3-5x^2+6x}-\dfrac{x-2}{x^4-x^3-6x^2}$ (h) $\dfrac{x}{x+2}+\dfrac{1}{x}-\dfrac{4}{x+1}$

(i) $1-\dfrac{1}{x+1}+\dfrac{2}{x^2-1}$ (j) $\dfrac{\dfrac{x}{x-2}-\dfrac{2x}{x^2-4}}{\dfrac{x-1}{x^2-9}+\dfrac{x}{x^2+5x+6}}$

(k) $\dfrac{5}{x^2+x-12}+\dfrac{4}{x^2-x-20}-\dfrac{3}{x^2-8x+15}$

9. Rationalize the denominators of the following.

(a) $\dfrac{5}{\sqrt{3}}$ (b) $\dfrac{\sqrt{2}}{4\sqrt{7}}$ (c) $\dfrac{1-\sqrt{3}}{2+\sqrt{3}}$ (d) $\dfrac{\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

(e) $\dfrac{5}{x\sqrt{x}}$ (f) $\dfrac{-7x^3}{\sqrt{x+1}}$ (g) $\dfrac{2x}{x+\sqrt{x+2}}$ (h) $\dfrac{7}{\sqrt{x+2}-\sqrt{x+1}}$

(i) $\dfrac{x^2}{\sqrt{x^2-1}+\sqrt{x+3}}$ (j) $\dfrac{\sqrt{x}}{\sqrt{x}-2}$ (k) $\dfrac{\sqrt{x}}{\sqrt{x}-2}$

10. Solve the following equations (i.e. find all real solutions).

(a) $3x^2-7x+1=2x^2-2x-5$ (b) $x^2-2x-2=0$ (c) $x^2-2x+2=0$

(d) $9x^2-30x+25=0$ (e) $x^3+27=0$ (f) $\dfrac{2}{x}-\dfrac{6}{x+1}=\dfrac{-5}{x+3}$

(g) $\dfrac{7}{x+2}-\dfrac{2}{x^2-4}=1$ (h) $\sqrt{3x+1}+1=x$ (i) $\sqrt{3x+1}=\sqrt{x}+3$

11. Solve the following systems of equations.

(a) $\left\{ \begin{array}{c} 3x-y=-1 \\ 3x-5y=-11 \end{array} \right\}$ (b) $\left\{ \begin{array}{c} 4x^2+y^2=68 \\ 2x+y=10 \end{array} \right\}$ (c) $\left\{ \begin{array}{c} x^2+xy=-1 \\ x-y+3=0 \end{array} \right\}$

(d) $\left\{ \begin{array}{c} y=x^2 \\ x+2y=2 \end{array} \right\}$ (e) $\left\{ \begin{array}{c} y=x \\ y=x^3 \end{array} \right\}$

Answers

1. (a) $x^{\frac{6}{5}}$ (b) $x^{\frac{3}{8}}$ (c) $x^{-\frac{5}{2}}$ (d) $x^{-\frac{4}{3}}$ (e) $x^{\frac{1}{12}}$ (f) $x^{-\frac{1}{10}}$

2. (a) $x^{\frac{11}{2}}$ (b) $x^{-\frac{22}{7}}$ (c) $x^{-\frac{12}{5}}$ (d) $x^{-\frac{19}{15}}$ (e) $x^{\frac{8}{27}}$ (f) $x^{-\frac{5}{2}}$ (g) $x^{\frac{2}{3}}$ (h) $x^{-\frac{3}{2}}$
(i) $x^{\frac{7}{3}}$ (j) $x^{\frac{3}{5}}$ (k) $x^{-\frac{2}{3}}$ (l) $x^{\frac{5}{4}}$

3. The true equations are (b), (c), (f), (i) and (j).

4. (a) $8x^3-3x+2$ (b) $-9x^3+4x^2+6x-10$ (c) $-x^4+2x^2-x+7$ (d) $3x^2-9x+4$
(e) $x^7-4x^5+2x^3-14x^2+5x+20$ (f) $2x^2-6x-8$ (g) $-8x^6+6x^4+24x^3+12x^2-8$

5. (a) $4x^2-12x+9$ (b) $x^3+15x^2+75x+125$ (c) $8x^3-36x^2+54x-27$
(d) $x^5-10x^4+40x^3-80x^2+80x-32$ (e) $16x^4+32x^3+24x^2+8x+1$

6. (a) $x^2(x-3)(x+3)$ (b) $(x-\sqrt{10})(x+\sqrt{10})$ (c) $-3x^2(x-\sqrt{7})(x+\sqrt{7})$
(d) $3x^3(x-2\sqrt{2})(x+2\sqrt{2})$ (e) $7x^4(x^2+4)$ (f) $(x-16)(x-1)$
(g) $-4x^2(x-6)(x+1)$ (h) $(3x-7)(2x-1)$ (i) $-2x(3x-4)(3x-2)$
(j) $(x^2+9)(x-3)(x+3)$ (k) $(x^2+3)(x-\sqrt{3})(x+\sqrt{3})$ (l) $(x^2+9)(x-2)(x+2)$
(m) $3x(x+1)(x-1)(x-\sqrt{5})(x+\sqrt{5})$ (n) $-2x^2(3x^2+1)(2x^2+7)$
(o) $(x+2)(x^2-2x+4)$ (p) $(2x-3)(4x^2+6x+9)$
(q) $(x+\sqrt[3]{3})[x^2-\sqrt[3]{3}x+(\sqrt[3]{3})^2]$ (r) $x^5(4x+3)(16x^2-12x+9)$
(s) $-3x^2\left(x-\sqrt[3]{\frac{4}{3}}\right)\left[x^2+\sqrt[3]{\frac{4}{3}}x+\left(\sqrt[3]{\frac{4}{3}}\right)^2\right]$

7. (a) $\dfrac{x+3}{x-3}$ $(x \neq -3)$ (b) $\dfrac{x}{4x+7}$ $(x \neq 0)$ (c) $\dfrac{x^2+5x+6}{x^2+5x}$

(d) $\dfrac{x^2-2x+4}{x-2}$ $(x \neq -2)$ (e) $\dfrac{-x-5}{x^2+2x}$ $(x \neq 2, x \neq 0)$

8. (a) $\dfrac{x-2}{x}$ (b) $\dfrac{4x^2+x-5}{3x^3+3x^2-18x}$ (c) $\dfrac{x+4}{x^2-3x+2}$ (d) $\dfrac{-4x^2+4x+3}{2x^2-x-6}$

(e) $\dfrac{6x^2-3x}{x^2-9}$ (f) $\dfrac{4x^2-7x+15}{x^3-3x^2+7x-5}$ (g) $\dfrac{x^3+2x^2+6x-4}{x^5-3x^4-4x^3+12x^2}$

(h) $\dfrac{x^3-2x^2-5x+2}{x^3+3x^2+2x}$ (i) $\dfrac{x^2-x+2}{x^2-1}$ (j) $\dfrac{x^4-9x^2}{2x^3-6x^2+2x+4}$

(k) $\dfrac{6x-49}{x^3-4x^2-17x+60}$

9. (a) $\dfrac{5}{3}\sqrt{3}$ (b) $\dfrac{\sqrt{14}}{28}$ (c) $5-3\sqrt{3}$ (d) $\dfrac{\sqrt{15}+3}{2}$ (e) $\dfrac{5\sqrt{x}}{x^2}$ (f) $\dfrac{-7x^3\sqrt{x+1}}{x+1}$

(g) $\dfrac{2x^2-2x\sqrt{x+2}}{x^2-x-2}$ (h) $7\sqrt{x+2}+7\sqrt{x+1}$ (i) $\dfrac{x^2\sqrt{x^2-1}-x^2\sqrt{x+3}}{x^2-x-4}$

(j) $\dfrac{x+2\sqrt{x}}{x-4}$ (k) $\dfrac{\sqrt{x^2-2x}}{x-2}$

10. (a) $\{2,3\}$ (b) $\{1+\sqrt{3}, 1-\sqrt{3}\}$ (c) $\varnothing$ (d) $\{\frac{5}{3}\}$ (e) $\{-3\}$ (f) $\{2,3\}$
(g) $\{3,4\}$ (h) $\{5\}$ (i) $\{16\}$

11. (a) $\{(\frac{1}{2}, \frac{5}{2})\}$ (b) $\{(4,2),(1,8)\}$ (c) $\{(-1,2),(-\frac{1}{2}, \frac{5}{2})\}$
(d) $\{(-\frac{1}{4}+\frac{\sqrt{17}}{4}, \frac{9}{8}-\frac{\sqrt{17}}{8}), (-\frac{1}{4}-\frac{\sqrt{17}}{4}, \frac{9}{8}+\frac{\sqrt{17}}{8})\}$ (e) $\{(0,0),(1,1),(-1,-1)\}$