UNIVERSITY OF MANITOBA

DATE: April 23, 2008
PAPER # 603
COURSE: MATH 1520
EXAMINATION: Intro. Calc. for Mgmt & Soc. Sci.

FINAL EXAMINATION

TIME: 2 hours
EXAMINER: Various

PART A: Multiple Choice Questions

Answer each question by putting one of the letters A, B, C, D, E in the appropriate answer box below.

Question #	1	2	3	4	5	6	7	8	9	10	11	12
Answer	B	D	B	D	D	D	A	D	B	A	C	A
Value	2	2	2	2	2	2	2	2	2	2	2	2

[2] 1. Let the supply and demand functions for a certain model of electric pencil sharpener be given by

$$p = S(q) = \frac{2}{3}q \quad \text{and} \quad p = D(q) = 16 - \frac{2}{3}q,$$

where p is the price in dollars and q is the quantity of pencil sharpeners (in hundreds). Find the equilibrium quantity and the equilibrium price.

A. Equilibrium quantity: 640, Equilibrium price: \$9.60

B. Equilibrium quantity: 1200, Equilibrium price: \$8.00

C. Equilibrium quantity: 950, Equilibrium price: \$7.00

D. Equilibrium quantity: 950, Equilibrium price: \$7.00

E. none of the above

need $S=D$

$\Rightarrow \frac{2}{3}q = 16 - \frac{2}{3}q$

$\Rightarrow \frac{4}{3}q = 16 \Rightarrow q = 12$

[2] 2. Let $C(x)$, $R(x)$ and $P(x)$ be the cost function, the revenue function and the profit function, respectively. If c is the break-even quantity, then which one of the following MUST be true?

A. $P'(c) = 0$

B. $P(c) = C(c) - R(c)$

C. $C'(c) = R'(c)$

D. $C(c) = R(c)$ (or $P(c) = 0$)

E. none of the above

[2] 3. If $2^{(8-2x)} = 16$ then x has the value:

A. -2

B. 2

C. 4

D. 8

E. none of the above

16 is 2^4

$\Rightarrow 2^{(8-2x)} = 2^4$

$\Rightarrow 8 - 2x = 4 \Rightarrow 2x = 4$

[2] 4. Lucky Smart invests a \$10,000 inheritance in a fund paying 9% per year compounded semi-annually. What will this investment be worth after 10 years?

A. $10,000e^{20}$

B. $10,000\,(1+0.09)^{20}$

C. $10,000\left(1+\dfrac{0.09}{2}\right)^{10}$

D. $10,000\left(1+\dfrac{0.09}{2}\right)^{20}$

E. none of the above

$A = 10000\left(1+\frac{0.09}{2}\right)^{20}$

[2] 5. Let $A > 0$, $B > 0$ and $1 \neq b > 0$. If $\log_b A = 2$ and $\log_b B = -4$ then the expression $\log_b \dfrac{A}{B}$ has the value:

A. -2

B. $-\dfrac{1}{2}$

C. $\dfrac{1}{2}$

D. 6

E. does not exist

$\log_b \frac{A}{B} = \log_b A - \log_b B$

$= 2 - (-4) = 6$

[2] 6. Consider the function $f(x) = |x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$.

Which of the following is NOT true?

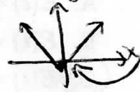
A. $f(0) = 0$ ✓

B. $\lim_{x\to 0} f(x) = 0$ ✓ $\left(\lim_{x\to 0^-} f(x) = 0 \;\&\; \lim_{x\to 0^+} f(x) = 0\right)$

C. f is continuous at $x = 0$ ✓ (b/c A. = B.)

D. f is differentiable at $x = 0$

E. $f(-x) = f(x)$ ✓

↳ recall $|x| =$  ← not differentiable at $x=0$

[2] 7. The following table lists the values of the functions f and g and their derivatives at several points.

x	1	2	3	4
$f(x)$	4	3	1	2
$f'(x)$	−1	2	−5	1
$g(x)$	3	4	1	2
$g'(x)$	−4	−6	−3	4

Find $\dfrac{d}{dx} g[f(x)]$ at $x = 2$.

A. -6

B. -3

C. 2

D. 30

E. not enough information provided

chain rule, $= g'(f(x)) \cdot f'(x)$

$= g'(f(2)) \cdot f'(2)$

$= g'(3) \cdot 2$

$= (-3)(2) = -6$

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[2] 8. The total revenue from the sale of x (in hundreds) stereos is given by $R(x) = 20 - 8x + 0.8x^2$. Find the marginal revenue when 1000 stereos are sold.

A. -8

B. 0

C. 8

D. 1592

E. none of the above

$R'(x) = -8 + 1.6x$

$R'(1000) = -8 + 1600$

[2] 9. Let $f(x) = 2008x^3 - 2009x^2 + 2010x - 2011$. Find the 28th derivative, $f^{(28)}(x)$.

A. -28

B. 0

C. 28

D. any positive real number

E. not enough information provided

[2] 10. Let $h(x) = \ln x$. Find the second derivative of $h(x)$ at $x = 2$, i.e., find $h''(2)$.

A. $-\frac{1}{4}$

B. 0

C. $\frac{1}{4}$

D. $\frac{1}{2}$

E. none of the above

$h'(x) = 1/x$

$h''(x) = -1/x^2$

$h''(2) = -1/2^2$

[2] 11. The rate at which an assembly line worker's efficiency E (expressed as a percent) changes with respect to time t is given by $E'(t) = 60 - 6t$, where t is the number of hours since the worker's shift began. Assuming that $E(0) = 35$, find $E(t)$.

A. $E(t) = 60t - 6t^2 + 35$

B. $E(t) = 60t - 3t^2 + 92$

C. $E(t) = 60t - 3t^2 + 35$

D. $E(t) = 60t - 3t^2 + 149$

E. none of the above

$E(t) = 60t - 6\,t^2/2 + C$

$= 60t - 3t^2 + C$

$E(0) = C = 35$

[2] 12. Let a be a real number. Which one of the following has a value of zero?

A. $\int_a^a x^2\,dx$

B. $\int_{-a}^a x^2\,dx$

C. $\int_0^a x^2\,dx$, where $a > 0$

D. $\int_a^0 x^2\,dx$, where $a < 0$

E. none of the above

PART B: Short Answer Questions

Answer all questions in the space provided. Only your answers will be marked.

[2] 13. State the formula for the definition of the derivative.

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

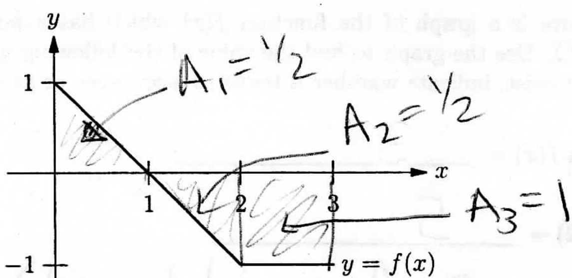
[2] 14. Write the equation which means that a function f is continuous at $x = c$.

$\lim_{x\to c} f(x) = f(c)$

[2] 15. State what limits need to be evaluated in order to determine if $y = a$ is a horizontal asymptote of a function $f(x)$.

$\lim_{x\to\pm\infty} f(x)$ (will $= a$ if a is an h.a.)

In questions 16–19 below, use the following graph of $y = f(x)$.



[2] 16. In the graph above, $\int_0^1 f(x)\,dx = \frac{1}{2}(1)(1) = \frac{1}{2}$

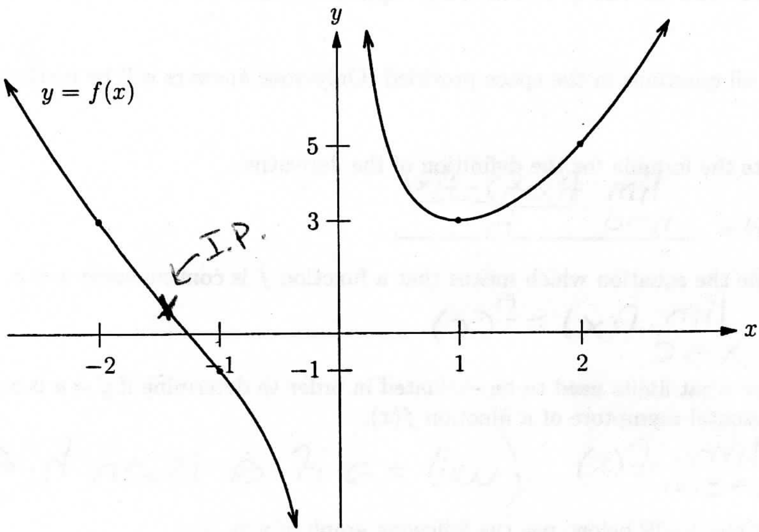
[2] 17. In the graph above, $\int_0^3 f(x)\,dx = \int_0^1 f(x)dx - \int_1^3 f(x)dx = \frac{1}{2} - (1+\frac{1}{2}) = 1$.

[2] 18. In the graph above, $\int_0^1 f'(x)\,dx = f(1) - f(0) = 0-1=-1$ (since $f(x)$ is the antiderivative of $f'(x)$)

[2] 19. In the graph above, $f'(1) = -1$.

↳ slope of the tangent line to $f(x)$ at $x=1$

20. Write your final answers for this question on the lines provided below.



The above is a graph of the function $f(x)$ which has a point of inflection at $x = -\sqrt{2}$. Use the graph to find the value of the following quantities. If a limit does not exist, indicate whether it tends to $+\infty$, $-\infty$, or neither.

[1] (a) $\lim_{x\to 1} f(x) =$ 3 .

[1] (b) $f(2) =$ 5 .

[1] (c) $f'(1) =$ 0 (horizontal tangent)

[1] (d) $\lim_{x\to 0^+} f(x) =$ ∞ .

[1] (e) Average rate of change of f between $x = -2$ and $x = -1$ is: $\dfrac{f(-1) - f(-2)}{-1-(-2)}$

$= \dfrac{-1-3}{1} = -4$.

Use the graph to state whether the sign of each of the following is " + " or " − ".

[1] (f) Sign of $f'(2) =$ + (inc) .

[1] (g) Sign of $f''(-1) =$ − (CD) .

[1] (h) Sign of $f''(1) =$ + (CU) .

21. Let $f(x) = 3x^5 - 10x^3 + 15x$. Then

$$f'(x) = 15(x-1)^2(x+1)^2 \quad \text{and} \quad f''(x) = 60x(x^2-1).$$

$= (x-1)(x+1)60x$

CD, CU, CD, CU
-1 0 1

always tve

[16] (a) Compile the following information about f and its graph. Give answers ONLY. (Write "none" if the item does not exist.)

domain? $(-\infty, \infty)$

y-intercept? 0

critical numbers of f? $x=1$ & $x=-1$

$f'(-2)= +$ -1 $f'(0)=+$ 1 $f'(10)=+$

open interval(s) where f is increasing? $(-\infty, \infty)$

open interval(s) where f is decreasing? never

open interval(s) where f is concave up? $(-1, 0)$ & $(1, \infty)$

open interval(s) where f is concave down? $(-\infty, -1)$ & $(0, 1)$

x- and y-coordinates of any local maxima? none

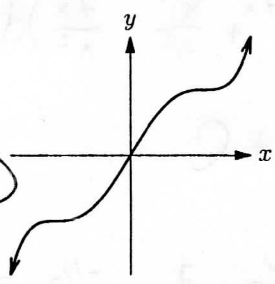
x- and y-coordinates of any local minima? none

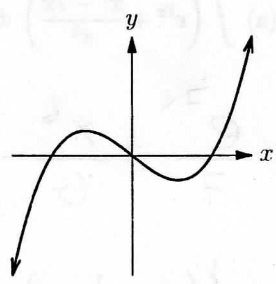
since always increasing

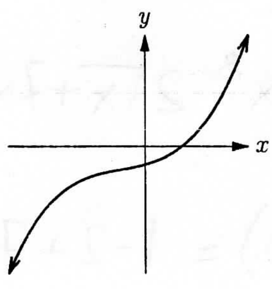
x-coordinate of any inflection points? $(-1,-8)$ & $(0,0)$ & $(1,8)$

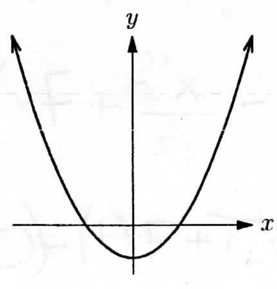
[2] (b) Given the information in part (a), choose which of the following graphs is the graph of $y = f(x)$. Put your answer on the blank line below.

(b) I

I.


II.


III.


IV.


PART C: Long Answer Questions

Please show your work clearly. If necessary, you may continue your work on the reverse sides of the pages, but please indicate clearly that your work is continued elsewhere.

22. Differentiate each of the following functions. **DO NOT SIMPLIFY YOUR ANSWERS.**

[4] (a) $y = 7^7 - \log_7 x + \sqrt[7]{x} - 7^x$ $= 7^7 - \log_7 x + x^{1/7} - 7^x$

$\Rightarrow y' = 0 - \frac{1}{x \cdot \ln 7} + \frac{1}{7} x^{-6/7} - 7^x \cdot \ln 7$

[7] (b) $y = \left(\frac{e^{-x}}{x^3+7}\right)^2$

quotient rule

$\Rightarrow y' = \underbrace{2\left(\frac{e^{-x}}{x^3+7}\right) \cdot \left(\overbrace{\frac{-e^{-x}(x^3+7) - 3x^2 \cdot e^{-x}}{(x^3+7)^2}}\right)}_{\text{chain rule}}$

23. Evaluate each of the following integrals. (Simplify your answers.)

[5] (a) $\int \left(e^{7x} + \frac{x^7 - 7x}{x^2}\right) dx$ $= \int \left(e^{7x} + \frac{x^7}{x^2} - \frac{7x}{x^2}\right) dx = \int \left(e^{7x} + x^5 - \frac{7}{x}\right) dx$

$= \frac{e^{7x}}{7} + \frac{x^6}{6} - 7 \ln|x| + C$

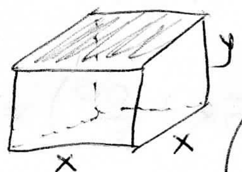
[7] (b) $\int_0^1 \left(8x^7 - \frac{1}{\sqrt{x}} + 7\right) dx$ $= \int_0^1 (8x^7 - x^{-1/2} + 7)\, dx$

$= \left(\cancel{8} \frac{x^8}{\cancel{8}} - \frac{x^{1/2}}{1/2} + 7x\right)\Big|_0^1 = (x^8 - 2\sqrt{x} + 7x)\Big|_0^1$

$= 1^8 - 2\sqrt{1} + 7(1) - (0 - 0 + 0) = 1 - 2 + 7 = 6$

24. A manufacturer requires a metal box with no top, a volume of 6 m^3, and a square base. The sides of the box cost \$2 per m^2, and the bottom of the box costs \$3 per m^2.

[6] (a) Determine the cost function for manufacturing the box by drawing a diagram and clearly defining your variables. Write your answer as a function of one variable only.



$V = x \cdot x\, y = x^2 y = 6$ (constraint)

$\text{Cost} = 3(x \cdot x) + 2(4 \cdot xy) = 3x^2 + 8xy$

$y = 6/x^2$

$\Rightarrow C = 3x^2 + 8x(6/x^2) = 3x^2 + \frac{48}{x}$

[6] (b) Determine the dimensions of the box which minimize the cost of production. Remember to justify why your answer is the absolute minimum.

Domain: $x \in (0, \infty)$

$C' = 6x - \frac{48}{x^2} = \frac{6x^3 - 48}{x^2} = \frac{6(x^3 - 8)}{x^2} \Rightarrow$ In domain, C.P. at $x = 2$ ($x^3 = 8$ for $x = 2$)

$\searrow \quad \nearrow$

0 $\quad C'(1) = -\quad$ 2 $\quad C'(10) = +$

MIN

So we should let $x = 2$m & $y = \frac{6}{4}$ or $\frac{3}{2}$ m

[2] (c) Determine the minimal cost of the box.

$C = 3(2)^2 + \frac{48}{2} = 3 \cdot 4 + 24 = \$36.$

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25. Albert sells custom-built Home Theatre PC's (HTPC's). He determines that the demand function for his HTPC's is

$$p = 2010 - 5x + \frac{100}{x}$$

where p is the price and x is the number of HTPC's he sells in one year. Albert's costs are

$$C(x) = 100 + 2x\ln x$$

[3] (a) Determine the revenue function $R(x)$ for Albert's HTPC's.

$R(x) = (\text{price}) \cdot x = \left(2010 - 5x + \frac{100}{x}\right)x$

$= 2010x - 5x^2 + 100$

[3] (b) Determine the profit function $P(x)$.

$P(x) = R(x) - C(x) = 2010x - 5x^2 + 100 - (100 + 2x\ln x)$

$= 2010x - 5x^2 - 2x\ln x$

[4] (c) Determine Albert's marginal profit function. (Simplify your answer.)

$\rightarrow = P'(x)$

$P'(x) = 2010 - 10x - \left(2 \cdot \ln x + \frac{1}{x} \cdot 2x\right)$

$= 2010 - 10x - 2\ln x - 2$

$= 2008 - 10x - 2\ln x$

[9] 26. Find all absolute extrema, if they exist, of the function $f(x) = x^4 - 8x^2 + 1$ on the interval $[-1, 2]$. Clearly state all values of x where they occur.

$f'(x) = 4x^3 - 16x = 4x(x^2 - 4) = 4x(x-2)(x+2)$

C.p.'s at $\cancel{x=-2}$, $x=0$, $x=2$ ← also an endpoint

reject (not in the interval)

$f(-1) = (-1)^4 - 8(-1)^2 + 1 = 1 - 8 + 1 = -6$

$f(0) = 0 - 0 + 1 = 1$ ← ABS MAX

$f(2) = 2^4 - 8(2^2) + 1 = 16 - 32 + 1 = -15$ ← ABS MIN

END OF EXAM