Midterm Formulas

Lines:

1. Slope $= m = \dfrac{y_2 - y_1}{x_2 - x_1}$
2. Slope – Intercept Form: $y = mx + b$
3. Point – Slope Form: $y - y_1 = m(x - x_1)$

Cost:

1. Cost $= C(x) =$ (marginal)x + fixed $= mx + b$
2. Revenue $= R(x) =$ (price)(# units) $= px$
3. Profit $= P(x) =$ revenue – cost $= R(x) - C(x)$

note: "marginal" cost, revenue, or profit represents the derivative of these functions

Properties of Functions:

1. Domain: the set of all possible values of the independent variable (x).
2. Range: the resulting set of all possible values of the dependent variable (y).
3. Vertical line test: if a vertical line drawn at any point of a graph crosses the function more than once, it is not a function.

Exponential Functions:

1. $a^0 = 1, a^{-n} = \dfrac{1}{a^n}$
2. $(a^m)(a^n) = a^{m+n}$
3. $\dfrac{a^m}{a^n} = a^{m-n}$
4. $(a^m)^n = a^{mn}$
5. $(ab)^m = (a^m)(b^m)$
6. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$
7. *if* $a > 0$ & $a \neq 1 \Rightarrow a^m = a^n$ *then* $m = n$
8. Change of Base (Exp's): $a^x = e^{(\ln a)x}$

Interest: (principle=P, yearly rate =r, compounded m times/year, # years=t)

1. Simple Interest: $I = Prt$

2. Compound Interest: $A = P\left(1 + \frac{r}{m}\right)^{tm}$

3. Continuous Compounding: $A = Pe^{rt}$

4. Effective rate: $r_E = \left(1 + \frac{r}{m}\right)^{m} - 1$ (compound interest)

$r_E = e^{r} - 1$ (continuous compounding)

Logarithmic Functions:

1. $f(x) = \log_a x \quad for\ x > 0$
2. $y = \log_a x \Leftrightarrow a^y = x$
3. $\log_{10} x = \log x$
4. $\log_e x = \ln x$
   i) $\ln e = 1$
   ii) $\ln 1 = 0$
5. $\log_a(xy) = \log_a x + \log_a y$
6. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
7. $\log_a(x^r) = r\log_a x$
8. $\log_a a = 1 \Rightarrow \log_a(a^r) = r\log_a a = r$
9. $\log_a 1 = 0$
10. Change of Base (Logs): $\log_a x = \frac{\log_b x}{\log_b a} = \frac{\ln x}{\ln a}$

Growth & Decay:

1. For initial amount y_0 at time $t = 0$, the amount present at some later time t is given by $y = y_0 e^{kt}$

$k > 0 \Rightarrow growth$

$k < 0 \Rightarrow decay$

2. Half - Life $T = \frac{-\ln 2}{k}$

Limits:

1. If p(x) is a polynomial, then $\lim_{x \to a} p(x) = p(a)$
2. *If* $\lim_{x \to a^-} f(x) = L$ *and* $\lim_{x \to a^+} f(x) = M$ *and* $L \neq M$, *then* $\lim_{x \to a} f(x)$ *does not exist*
3. *If* $f(x) = \frac{p(x)}{q(x)}$ with p and q polynomials, then to take $\lim_{x \to \infty} f(x)$ divide p & q by the highest power of x in q (the denominator)

Continuity:

A fcn' f(x) is continuous at a point x = c if:

1. f(c) is defined
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

The Derivative:

1.The derivative represents the slope of the tangent line to a curve f(x)

2. Definition of the Deriative: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

3. The derivative does not exists at the places on a graph where there are sharp points, where the function is discontinuos, or where the tangent line is vertical.

Techniques for Derivatives:

1. Constant Rule: $f(k) = k \Rightarrow f'(x) = 0$
2. Power Rule: $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
3. Product Rule: $f(x) = u(x)v(x) \Rightarrow f'(x) = u'(x)v(x) + v'(x)u(x)$
4. Quotient Rule: $f(x) = \frac{u(x)}{v(x)} \Rightarrow f'(x) = \frac{u'(x)v(x) - v'(x)u(x)}{[v(x)]^2}$
5. Chain Rule: $y = f[g(x)] \Rightarrow y' = f'[g(x)] \cdot g'(x)$
   (derivative of the outside times derivative of the inside)