Chapter 9: Multivariable Calculus

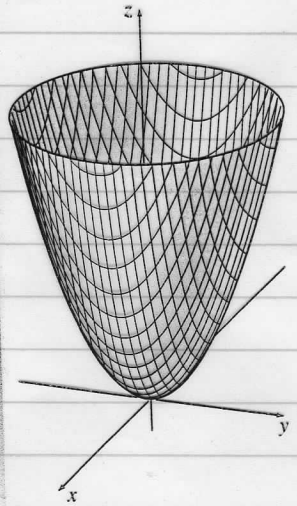
Section 9.1: Functions of Several Variables

Up to now, we have been working with functions of 1 variable, ie, $y = f(x)$. The same techniques we have learned so far for functions of 1 variable can be used for functions of more than one variable.

Fcn' of 1 variable	Fcn' of 2 variable
$y = f(x)$	$z = f(x, y)$
2-D (x, y)	3-D (x, y, z)
$y \rightarrow$ dependent var.	$z \rightarrow$ dependent var.
$x \rightarrow$ independent var.	$x, y \rightarrow$ independent var.
set of all possible x's $\rightarrow$ domain	set of all possible x & y's $\rightarrow$ domain
resulting set of y's $\rightarrow$ range	resulting set of z's $\rightarrow$ range

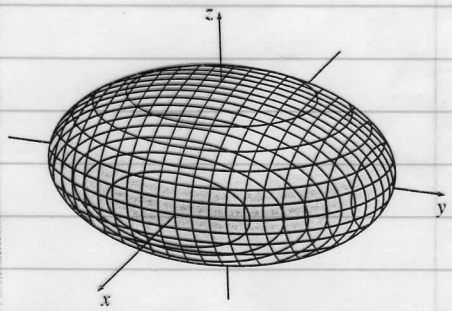
Paraboloid, $z = x^2 + y^2$

xy-trace: point
yz-trace: parabola
xz-trace: parabola



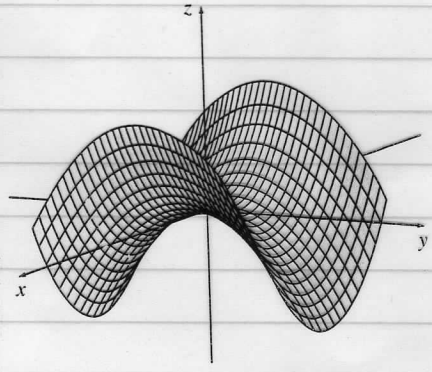
Ellipsoid, $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

xy-trace: ellipse
yz-trace: ellipse
xz-trace: ellipse



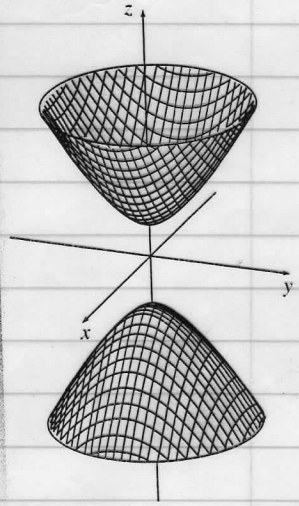
Hyperbolic Paraboloid, $x^2 - y^2 = z$
(sometimes called a *saddle*)

xy-trace: two intersecting lines
yz-trace: parabola
xz-trace: parabola



Hyperboloid of Two Sheets, $-x^2 - y^2 + z^2 = 1$

xy-trace: none
yz-trace: hyperbola
xz-trace: hyperbola



Section 9.2: Partial Derivatives

Recall: for the one-variable function $y = f(x)$, we had the notation $\frac{dy}{dx} = f'(x)$ for a derivative, where $\frac{dy}{dx}$ tells us that we are looking for the derivative of the function y, with respect to (w.r.t.) x.

For multivariable fcn's, we can also find the derivative, the only difference is that we now have to specify with respect to which variable we are taking the derivate.

We call these derivatives "partial derivatives" because they only find the derivative w.r.t. part of the function's variables, not all.

Higher Order Partial Derivatives: Just as we could find $f''(x)$, $f'''(x)$, $f^{(4)}(x)$, ..., we can find higher order partial derivatives. However, a one-variable function has only one of each of the higher order derivatives. Multi-variable functions have much more.