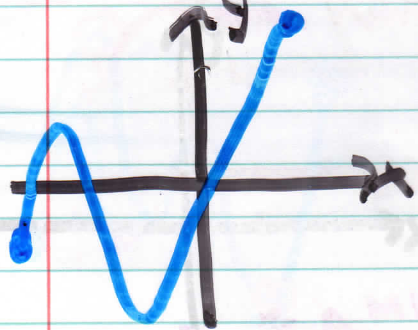
Chapter 6 : Applications of the Derivative

Section 6.1 : Absolute Extrema

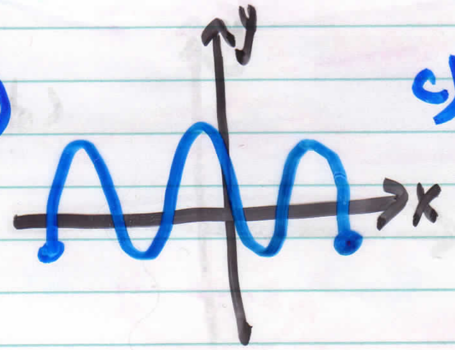
In section 5.2, we used $f'(x)$ to find the relative extrema of $f(x)$. In some cases, we sew that $f(x)$ has more than one relative max or min. The highest of these relative max's is the "absolute max" & the lowest relative min. is the "absolute min".

ex// label the absolute max (a.max) & absolute min (a. min) in the following sketches:

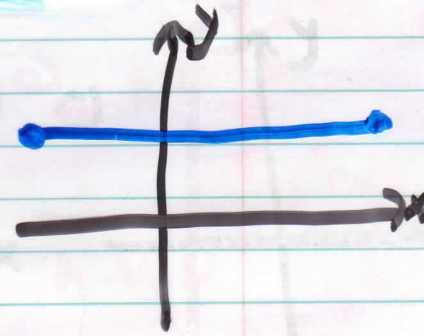
a)



b)



c)



Notice that in b) & c), although the function has only one absolute. max value & one abs min value, this value can occur at more than one point.

Extreme Value Theorem: A function f that is continuous on a closed interval [a, b] will have both an absolute max & an absolute min. on the interval (which could be at the end points).

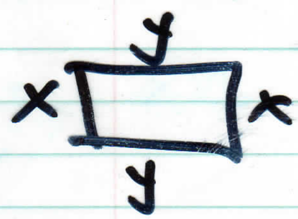
Section 6.2: Applications of Extrema

In many real life problems, we need to maximize something (like profit) or minimize something (like cost). The last section and our knowledge of functions (such as profit, cost, area, perimeter, etc.) can give us the tools to solve such max/min problems.

To solve an applied extrema problem, we follow these steps:

1) Read the problem carefully, write down what is known & what needs to be found *
2) Make a sketch & label it with info. from 1) *
3) Decide which variable (equation) must be maximized or minimized (f)
4) Use information given to write the above equation (f) as a function of only one variable.
5) Find the domain of f
6) Find the critical points of f.
7) Evaluate CP's (& endpoints) (plug into f) to find your absolute extrema

Formulas to remember:



Perimeter = add lengths of each side

Area = (length)(width), surface area = add all areas

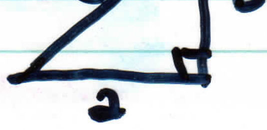
Volume = (length)(width)(height)

Area of Triangle = $\frac{1}{2}$(base)(height)

Area of Circle = πr^2 (r = radius)

Circumference of Circle = $2\pi r$

Pythagoras ⇒ $a^2+b^2=c^2$

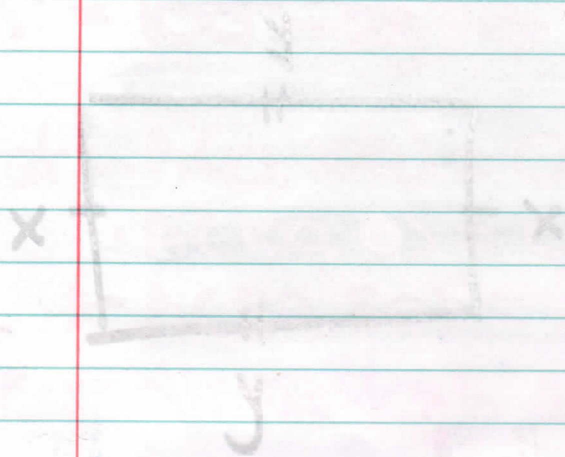


ex// The sale of CD's of "lesser" performers is very sensitive to price. If a CD manufacturer charges $p(x)$ dollars per CD, where $p(x) = 6 - \frac{x}{8}$, then x thousand CD's will be sold.

a) find an expression for the total revenue from the sale of x thousand CD's.

b) find the value of x that leads to maximum revenue & find this revenue.

ex// Find the dimensions of the rectangular field of maximum area that can be made from 200 m of fencing material.



Area = $xy = 20,000$

Minimize cost

$C = 3y + 6x + 3y + 6x$

$C = 6y + 12x$

ex// A fence must be built to enclose a rectangular area of 20,000 ft². Fence material costs \$3 per foot for the two sides facing north & south, \$6 per foot for the other 2 sides. Find the cost of the least expensive fence.

$y = \frac{20000}{x}$ $\quad C = 6\left(\frac{20,000}{x}\right) + 12x$

$C = \frac{120,000}{x} + 12x$

$= 120000x^{-1} + 12x$

Domain: $(0, \infty)$

$C' = -120,000x^{-2} + 12 = \frac{-120,000}{x^2} + 12$

$= \frac{-120,000 + 12x^2}{x^2}$

$C' = 0$ for $12x^2 = 120,000 \Rightarrow x^2 = 10,000$

$x = \pm\sqrt{10000} = \pm 100$

ex// A television manufacturing firm needs to design an open-topped box with a square base. The box must hold 32 in^3. Find the dimensions of the box that can be built with the minimum amount of materials.

ex// A cylindrical box will be tied up with ribbon. The longest ribbon available is 130 cm long & 10 cm are required for the bow. Find radius & height of a box with largest possible volume