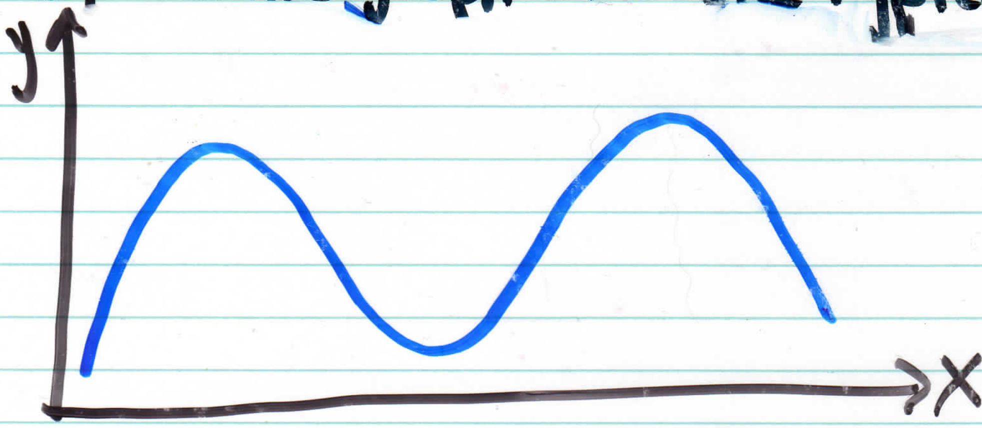
Chapter 5: GRAPHS & THE DERIVATIVE

Section 5.1: Increasing/Decreasing Fcn's

If we draw the graph of some typical fcn'



we can see that it increases in some areas & decreases in others. We can tell from just the equation of a fcn' the regions of increase & decrease.

?How?

Increasing/Decreasing: if $f(x)$ has a derivative,

1. if $f'(x) > 0$ for each x in an interval; $f(x)$ is increasing
   - (ie, $f(x_1) < f(x_2)$ for $x_1 < x_2$)
2. if $f'(x) < 0$ for each x in an interval, $f(x)$ is decreasing
   - (ie, $f(x_1) > f(x_2)$ for $x_1 < x_2$)
3. if $f'(x) = 0$ for each x in an interval, $f(x)$ is constant.

? How do we find these intervals ?

The derivative changes signs (from +ve to -ve or -ve to +ve) at points where

$f'(x) = 0$ & where $f'(x)$ d.n.e.

So we find the "critical numbers" $x = c$ in the domain of f where $f'(c) = 0$ or $f'(c)$ d.n.e.

In summary, our test is as follows:

Section 5.2: Relative Extrema

If c is a number in the domain of a fcn' f, then $f(c)$ is a "relative maximum" if there exists an open interval (a,b) containing c such that $f(x) \le f(c)$ for all x in (a,b), & $f(c)$ is a "relative minimum" if $f(x) \ge f(c)$ for all x in (a,b).

note: if a fcn' has either a relative min. or relative max., it is said to have a "relative extrema".

A relative extrema can be either a critical point or a domain endpoint, but

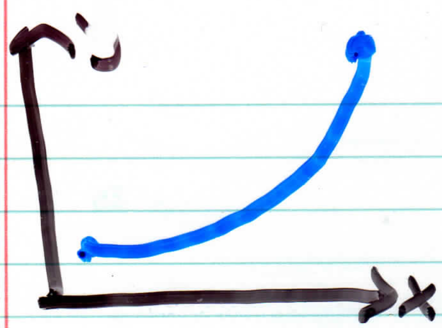
? How do we test these points to see if they are a relative max or min?

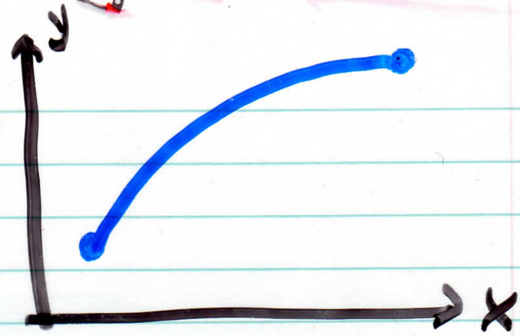
Section 5.3 : Higher Derivatives, Concavity, & the Second Derivative Test

We know that $f'(x)$ can tell us about a fcn' $(f(x))$'s increasing & decreasing. So say we are investing in a product whose profit fcn' is $P(x)$. If $P'(x) > 0$, we know that profit is always increasing. However, whether or not this is a good investment depends on the *rate of increase*. This rate is $P''(x)$.

Higher Derivatives: You can also take the derivative of a derivative

Look at the following 2 functions:

1 

2 

Concavity Test:

a) if $f''(x) > 0$ for all x in I, f is CU on I.

b) " $f''(x) < 0$ " " " ", " " CD ".

Possible shapes of graphs based on f' & f"

f" \ f'	+, +	+, -	-, +	-, -
+ CU				
- CD				

ex// sketch a possible graph of a fcn f that would satisfy the following:

i) $f'(x) > 0$ on $(-\infty, 1)$ & $f'(x) < 0$ on $(1, \infty)$

ii) $f''(x) > 0$ on $(-\infty, -2)$ & $(2, \infty)$, & $f''(x) < 0$ on $(-2, 2)$

iii) $\lim_{x \to -\infty} f(x) = -2$ & $\lim_{x \to \infty} f(x) = 0$

ex// find where $f(x)$ is concave up (CU) or concave down (CD) & find the inflection points.

$f(x) = x^4 - 8x^3 + 18x^2$

The second derivative can also tell us if a critical point $x=c$ (where $f'(x)=0$) is a relative max or min.

1) $f''(c) > 0$, $f(c)$ is a relative min

2) $f''(c) < 0$, $f(c)$ is a relative max

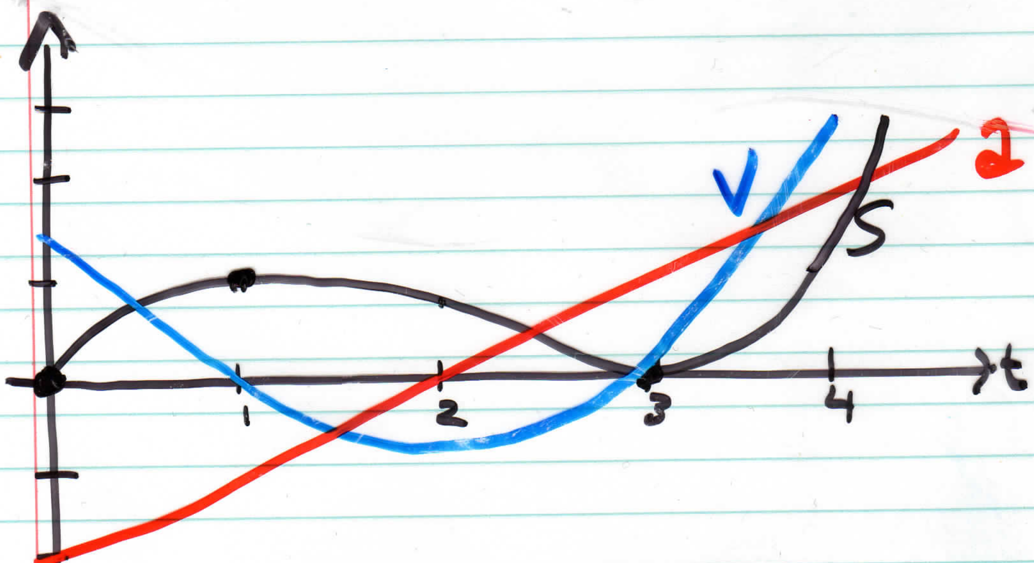
3) $f''(c) = 0$ (or d.n.e.) no info about extrema

In general, we can interpret the second derivative as a rate of change of a rate of change. If the 1st derivative velocity, then the 2nd derivative is acceleration, $a(t)$.

$$a(t) = v'(t) = s''(t)$$

ex// The position of a particle is given by the equation $s = f(t) = t^3 - 6t^2 + 9t$ (t - seconds, s - meters)

Find the acceleration at time t. What is the acceleration after 4 seconds?



Section 5.4: Curve Sketching

1. Domain: check where $f(x)$ is undefined
2. Intercepts: y intercept → set $x=0$
   x intercept → set $y=0$
3. Symmetry (optional): check if $f(-x) = \underbrace{f(x)}_{\text{even}}$ or $\underbrace{-f(x)}_{\text{odd}}$
4. Asymptotes: H.A. → check $\lim_{x\to\pm\infty} f(x)$ for $y=$ H.A.
   V.A. → check $\lim_{x\to a} f(x) = \infty$ for $x=$ V.A.
   ↳ places where $f(a)$ is undefined.
5. Inc/Dec: Compute f' & find the intervals where $f'(x)>0$ (INC) & $f'(x)<0$ (DEC), & critical numbers where $f'(x)=0$ or $f'(x)$ d.n.e.
6. Local Max/Mins: Check your critical numbers from 5 to see if they are max/mins.
7. Concavity: Compute f'' & find the possible inflection points. Check where $f''(x)>0$ (cu) & $f''(x)<0$ (cd) & confirm inflection points.
8. Sketch!! label important points first.

ex// Sketch the graph of a single function that has all of the following properties

a) continuous everywhere except at $x = -4$, where there is a vertical asymptote.

b) a y-intercept at $y = -2$

c) x-intercepts at $x = -3, 1, \& 4$

d) $f'(x) < 0$ on $(-\infty, -5), (-4, -1), \& (2, \infty)$ DEC.

e) $f'(x) > 0$ on $(-5, -4) \& (-1, 2)$ INC.

f) $f''(x) > 0$ on $(-\infty, -4) \& (-4, -3)$ CU.

g) $f''(x) < 0$ on $(-3, -1) \& (-1, \infty)$ CD.

h) differentiable everywhere except at $x = -4$ & $x = -1$. V Y