Chapter 4: Calculating The Derivative

Section 4.1: Techniques for Derivatives

Previously, we found $f'(x)$ using the definition, that is; $\boxed{f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}.}$

But, there are faster ways to find $f'(x)$.

There are also various way to write the derivative.

For $y = f(x)$:

① Constant Rule: $f(x) = k$ (some constant)

② Power Rule: $f(x) = x^n$

Section 4.2: Derivatives of Products & Quotients

We previously had a rule that if $f(x) = u(x) \pm v(x)$, then $f'(x) = u'(x) \pm v'(x)$. So if $f(x) = u(x) \cdot v(x)$, does $f'(x) = u'(x) \cdot v'(x)$? or if $f(x) = \frac{u(x)}{v(x)}$, does $f'(x) = \frac{u'(x)}{v'(x)}$?

③ **Product Rule:** $f(x) = u(x) \cdot v(x)$

④ **Quotient Rule:** $f(x) = \frac{u(x)}{v(x)}$

Marginal Cost: We noted before that the marginal cost fcn' is the derivative of the cost fcn' $C(x)$ (Marginal cost $= C'(x)$). So similarly:

Marginal Revenue $= R'(x) =$ price (x)

Marginal Profit $= P'(x) = (R(x) - C(x))' = R'(x) - C'(x)$

ex: If the cost fcn' is $C(x) = 2x^3 + x + 9$ & $R(x) = 5x^3 + x$, find the average cost fcn', marginal average cost fcn', & marginal profit fcn'.

a)

b) What is the marginal average profit on 10 units?

Section 4.3: The Chain Role

Composite Functions: if f & g are function, the "composite function" of g & f is $g(f(x))$ [or $f(g(x))$]

Section 4.4: Derivatives of Exponential Functions

1. $\frac{d}{dx}(e^x) = e^x$

2. $\frac{d}{dx}(a^x) = a^x \cdot \ln a$

3. $\frac{d}{dx}\left(e^{g(x)}\right) = e^{g(x)} \cdot g'(x)$

4. $\frac{d}{dx}\left(a^{g(x)}\right) = a^{g(x)} \cdot g'(x) \cdot \ln a$

Section 4.5: Derivatives of Logarithmic Functions

Say we have a fcn' $f(x) = \log_a g(x)$,

then $\underline{f'(x)} = ?$

1. $\frac{d}{dx}\left(\log_a g(x)\right) = \frac{g'(x)}{g(x) \cdot \ln a}$

2. $\frac{d}{dx}\left(\log_a x\right) = \frac{1}{x \cdot \ln a}$

3. $\frac{d}{dx}\left(\ln g(x)\right) = \frac{g'(x)}{g(x)}$

4. $\frac{d}{dx}\left(\ln x\right) = \frac{1}{x}$

Mid-Term Review

A firm producing electrical gadgets finds the total cost of producing & selling x units is given by $C(x) = 50x + 230$. Management plans to charge \$52 per gadget.

a) How many gadgets must be sold to Break even?

$C(x) = R(x)$

$50x + 230 = 52x \Rightarrow 230 = 2x$

$x = 230/2 \Rightarrow 115$ gadgets

b) What is the profit if 150 gadgets are sold?

$P(x) = R(x) - C(x) = 52x - 50x - 230$

$= 2x - 230$

$P(150) = 2(150) - 230 = 300 - 230 = \$70$

c) How many units must be sold to produce a Profit of \$10,000?

$P(x) = 10,000 = 2x - 230$

$2x = 10,230 \Rightarrow x = 5,115$ gadgets

d) What is the average cost per item if 50 are sold?

Ave Cost: $\dfrac{C(x)}{x} = \dfrac{50x + 230}{x}$

Ave cost of 50: $\dfrac{50(50) + 230}{50} = \$54.60$

Suppose the total cost per day $C(x)$ for manufacturing x windsurfer boards is given by $C(x)=3+40x-x^2$. Find the marginal cost fcn!

Marg. Cost $= C'(x) = 40-2x$

Assume the price p in dollars is given by $p=8000-20x$. Find the marginal revenue when demand is 100 units

$R(x) = \text{price}\,(\#\text{units}) = (8000-20x)\,x$

$= 8000x - 20x^2$

Marg. $R(x) = R'(x) = 8000-40x$, $R'(100) = 4000$

Suppose \$2000 is invested at 5% compounded annually. How long will it take for this investment to be worth \$8000?

$A = P(1+\frac{r}{n})^{nt} \Rightarrow 8000 = 2000(1+\frac{0.05}{1})^{1\cdot t}$

$4 = (1.05)^t \Rightarrow \ln 4 = \ln(1.05)^t$

$\ln 4 = t\ln(1.05)$, $t = \dfrac{\ln 4}{\ln(1.05)}$ //

If \$10,000 grows to \$10,400 when it is invested for 10 years at an annual rate r compounded continuously, what is the value of r?

$A = Pe^{rt} \Rightarrow 10{,}400 = 10{,}000e^{r\cdot 10}$

$\dfrac{10{,}400}{10{,}000} = e^{10r} \Rightarrow \ln\left(\dfrac{10{,}400}{10{,}000}\right) = \ln e^{10r}$

A population of 15,000 people grew exponentially to 17,000 in 4 years. Write an exponential equation for the population as a fcn' of time.

At this rate, how long will it take for the pop. to reach 45,000?

Evaluate the following limits (or state if they do not exist).

a) $\lim\limits_{x \to 81} \dfrac{\sqrt{x}-9}{x-81}$

b) $\lim\limits_{x \to 2} \dfrac{5x^2-12x+4}{x^2-4}$

c) $\lim\limits_{x \to -\infty} \dfrac{8x^3+6}{3x^3-1}$

Use the definition of the derivative to find $f'(x)$ given $f(x) = \frac{2}{x}$

Find the eqn' of the tangent line to the curve $f(x) = x^3 - 3x^2 - 6x + 1$ at $x = 2$.

Find all points x where the slope of the tangent line to the curve at x is -9.

Find $f'(x)$ for each of the fcn's below,

DO NOT SIMPLIFY

a) $f(x) = x^e + 5x^3 + \pi^3$

b) $f(x) = (\sqrt{x} + 10)(1+2x)$

c) $f(x) = \dfrac{3x^2+2x+1}{x^2+2x+3}$

d) $f(x) = \sqrt{4x^3 - x^2}$

e) $\sqrt[5]{x - \sqrt{x}} = f(x)$