ex/ Find the effective rate corresponding to each stated rate.
"nominal"

a) 6% compounded quarterly

b) 6% compounded continuously

Chapter 3: THE DERIVATIVE

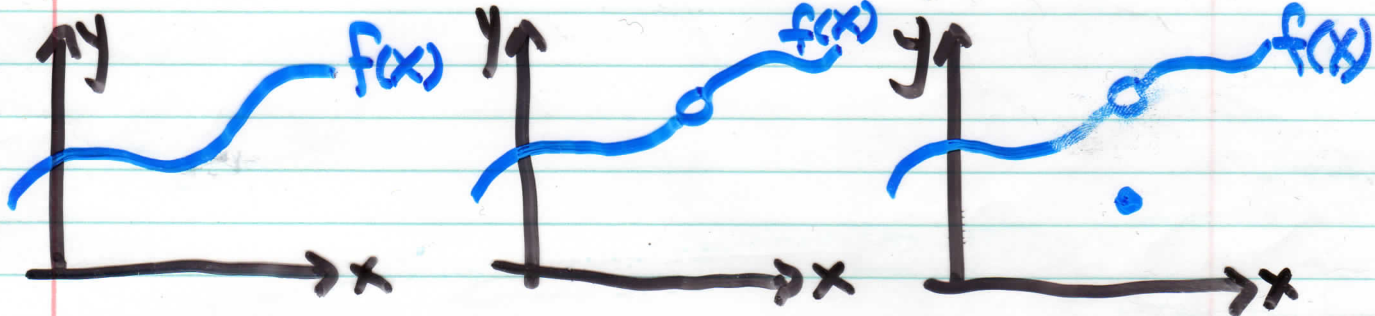
Section 3.1: Limits

What is the value of the fcn' $\frac{x-1}{x^2-1} = f(x)$ at $x=1$?

Defn: $\lim_{x \to a} f(x) = L$ / the limit of f(x) as x approaches a equals L

if we can make the values of f(x) arbitrarily close to L by taking x to be close to a (on either side) but NOT EQUAL to a.

ex//



ex// The Heaviside fcn' H is $H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$

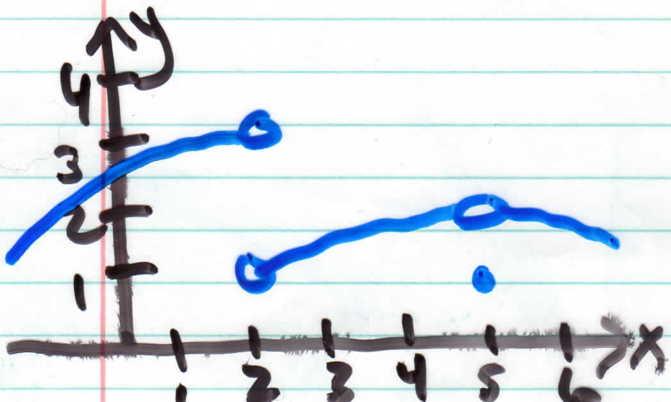
Defn': ① the limit of $f(x)$ as x approaches a from the left is written $\lim_{x \to a^-} f(x) = L$.

② the limit of $f(x)$ as x approaches a from the right is written $\lim_{x \to a^+} f(x) = L$.

Theorem': $\lim_{x \to a} f(x) = L$ if and only if

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$$

ex// Use the graph of g to state the values (if they exist) of the following:



a) $\lim_{x \to 2^-} g(x) =$

b) $\lim_{x \to 2^+} g(x) =$

c) $\lim_{x \to 2} g(x) =$

d) $\lim_{x \to 5^-} g(x) =$

e) $\lim_{x \to 5^+} g(x) =$

f) $\lim_{x \to 5} g(x) =$

g) $f(5) =$

ex// Sketch the graph of the following & use it to determine the values of a for which $\lim_{x \to a} f(x)$ exists:

$$f(x) = \begin{cases} 2 - x & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 1 \\ (x-1)^2 & \text{if } x \geq 1 \end{cases}$$

Infinite Limits: For some functions, the values of $f(x)$ get very large (+ or −) at some values of x.

ex// find $\lim_{x \to 0} \frac{1}{x^2}$ if it exists.

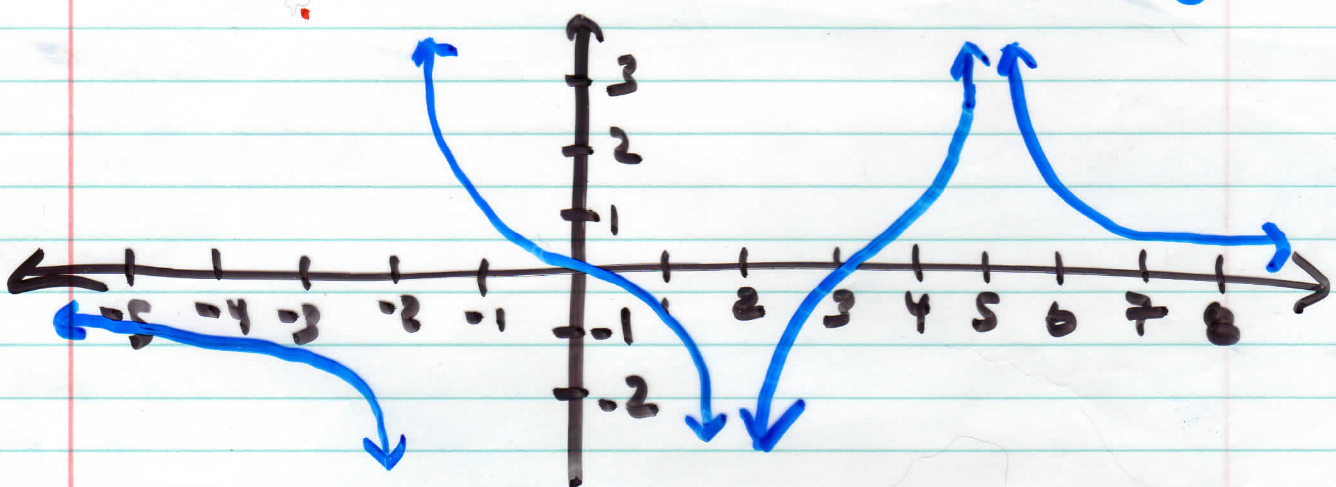
Defn': Let f be a fcn' defined on both sides of a, except at a itself, then $\lim_{x \to a} f(x) = \infty \ (-\infty)$ means that the values of f(x) can be made arbitrarily large (+ or −) by taking x close to a, but not equal to a.

ex// determine the infinite limits

Defn': the line $x=a$ is called a "vertical asymptote" (v.a.) if at least ONE of the following is true:

$$\lim_{x\to a} f(x) = \pm\infty, \quad \lim_{x\to a^-} f(x) = \pm\infty, \quad \lim_{x\to a^+} f(x) = \pm\infty$$

ex// For $R(x)$, state the following:



Calculating Limits Using Limit Laws:

In the last class, we used tables of values & graphs to guess limits, we can calculate **exact** limits with the *Limit Laws.*

ex// Use the Limit Laws & the graph to evaluate:

LIMIT LAWS: Suppose c is a constant and

a) $\lim_{x \to a} f(x) = A$ & b) $\lim_{x \to a} g(x) = B$ exist, then:

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = A + B$
2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = A - B$
3. $\lim_{x \to a} [c f(x)] = c \lim_{x \to a} f(x) = cA$
4. $\lim_{x \to a} [f(x) g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = A \cdot B$
5. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{A}{B}$ (provided $B \neq 0$)
6. $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n = A^n$
7. $\lim_{x \to a} c = c$
8. $\lim_{x \to a} x = a$
9. $\lim_{x \to a} x^n = a^n$
10. $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$ (n positive)
11. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} = \sqrt[n]{A}$ (n positive)

Direct Substitution Property: If f is a polynomial or rational fcn' & a is in the domain of f, then:

$$\lim_{x \to a} f(x) = f(a)$$

Limits at Infinity:

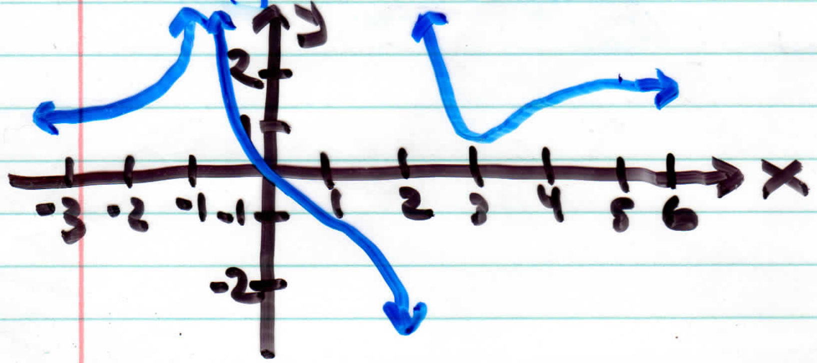
We know what a vertical asymptote is & how to find one: ($\lim_{x \to a} f(x) = \pm\infty$)

Now, we let x become larger & larger (+ or −) & see what happens to y.

For example, what happens to $\frac{1}{x}$ as $x \to \pm\infty$?

Defn: The line $y = L$ is called a "horizontal asymptote" (h.a.) if either $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to -\infty} f(x) = L$.

ex/ Find the infinite limits (v.a.'s) & limits at infinity (h.a.'s) of:



Theorem: If $r > 0$ is a rational number, then

$$\lim_{x\to\infty} \frac{1}{x^r} = 0 \qquad \left(\text{"}\frac{1}{\infty} = 0\text{"}\right)$$

If $r > 0$ is a rational number such that x^r is defined for all x, then $\lim_{x\to -\infty} \frac{1}{x^r} = 0$