Chapter 2: NONLINEAR FUNCTIONS

Section 2.1: Properties of Functions

Defn': a "function" is a rule that assigns each element from one set exactly one element from another by an equation (ie, one unit of feed has ~~one~~ cost, $ (20)

Defn': the "domain" is the set of all possible values of the indep. variable (x)

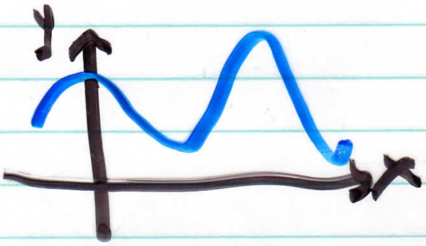
Defn': the "range" is the resulting set of all possible values of the dependent var. (y) note: Not all curves in the x-y plane are the graphs of functions!

A curve in the x-y plane is the graph of a fcn' $f(x)$ of x if and only if NO vertical line intersects the curve more than once.

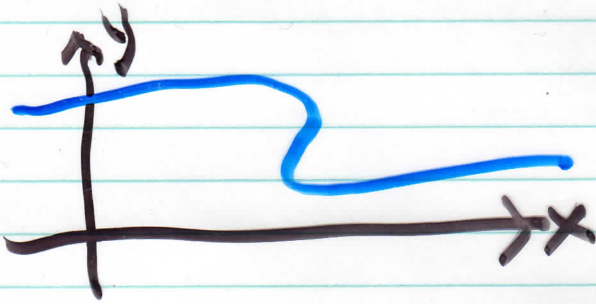
This is the "vertical line test".

ex// which of the following are functions?

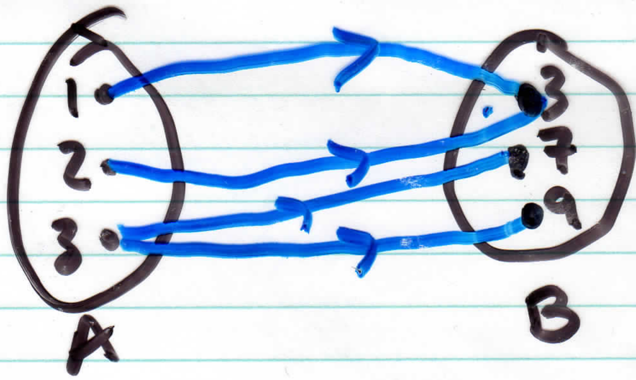
a)



b)



c)

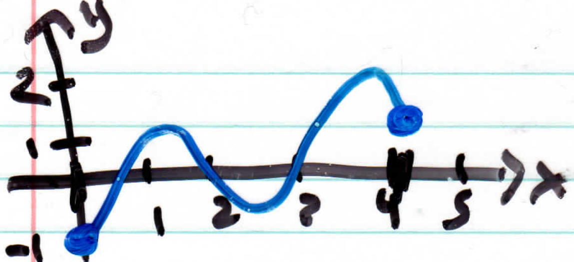


d) $x = y^2$

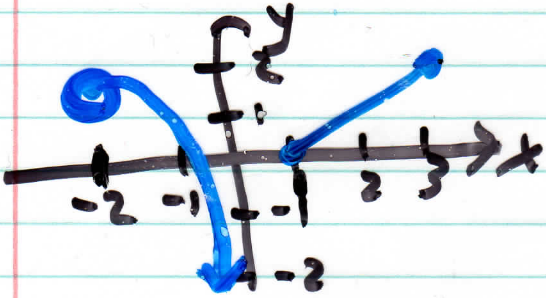
Type of Fcn'	Example	Domain
Polynomial Fcn $p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \dots$		all x, $(-\infty, \infty)$ $x \in \mathbb{R}$
Rational Fcn' $\frac{p(x)}{q(x)}$ $p(x) \leftarrow$ polyn. $q(x) \leftarrow$ polyn.		all x values except $q(x) = 0$
Root Fcn' $\sqrt{p(x)}$		$p(x) \geq 0$

ex// State the domain ($ range for a-c) of:

a)



b)



c) $g(x) = x^2$

d) $h(x) = \frac{3}{x^2 - 2x}$

e) $f(x) = \sqrt{x^2 + x - 20}$

<u>Piecewise Defined Fcn's</u>: Some fcn's are defined by different formulas in different parts of their domains.

ex// Find the domain & sketch $f(x) = \begin{cases} 2x+3 & \text{if } x < -1 \\ 3-x & \text{if } x \geq -1 \end{cases}$

<u>Increasing/Decreasing</u>: A fcn' f is "increasing" on an interval I if $f(x_1)<f(x_2)$ for $x_1<x_2$ in I. It is "decreasing" on I if $f(x_1)>f(x_2)$ whenever $x_1<x_2$ in I.

<u>Symmetry</u>: There are 2 types of symmetries that we discuss when we talk about fcn's.

1) <u>EVEN</u> fcn's: these satisfy $\boxed{f(-x)=f(x)}$ & are symmetric about the y-axis.

ex// $f(x)=x^2$

2) ODD fcn's: these satisfy $\boxed{f(-x) = -f(x)}$
& are symmetric about the origin (180°).

ex// $f(x) = x^3$

ex// Are the following fcn's odd, even, or neither?

Section 2.4: Exponential Functions

Def'n: an "exponential fcn" w/ base a is

$\boxed{f(x) = a^x}$ where $a > 0$ & $a \neq 1$

These types of fcn's are used to describe growth & decay ex// Solve the following

Compound Interest: when interest is paid (or charged) on interest as well as principle.

P = pricipal, r = rate (% per yer), t = time (years), I = interest

1) **Simple Interest**:

2) **Compound**:

If P dollars is invested at a yearly rate of interest of r per year, compounded m times/year for t years, the MEMORIZE "compound amount" is $\boxed{A = P\left(1+\frac{r}{m}\right)^{tm}}$

ex// A man earns \$1000 interest on an investment making 6% annual interest compounded quarterly for 2 years. How much did he invest?

e: as m becomes larger & larger, $(1+\frac{1}{m})^m$ gets closer & closer to e, approximately 2.71828182....

Continuous Compounding: if a deposit of P dollars is invested at a rate of interest r compounded continuously for t years, the compound amount is

$$\boxed{A = Pe^{rt}}$$

? How?

ex// Assuming continuous compounding, what will it cost to buy a \$10 item in 3 years at the following inflation rates?

Logarithmic Fcn's:

The "logarithmic fcn with base a" (where $a>0$ & $a\neq 1$) is $\boxed{\log_a x}$

$$\boxed{\log_a x = y \iff a^y = x}$$

Cancellation eqn's:

$\log_a(a^x) = x$ for $x \in (-\infty, \infty)$

$a^{\log_a x} = x$ for every $x \in (0, \infty)$

Logarithm Laws:

If x & y are positive numbers, then:

1. $\log_a(xy) = \log_a x + \log_a y$
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_a(x^r) = r\log_a x$

ex// evaluate $\log_2 80 - \log_2 5$

Of all possible bases for log, the base e is the most convenient. We call $\log_e x = \ln x$ the "natural logarithm"

We use logarithms to solve for variables in an exponent.

ex// solve for x:

Section 2.6: Applications in Growth & Decay

Defn': If y_0 is the amount of some quantity present at any time $t=0$, then the amount present at any later time t is given by the "exponential growth & decay fcn'"

$$\boxed{y = y_0 e^{kt}}$$

ex// Carbon 14 is a radioactive form of carbon that is found in all living organisms. After a plant or animal dies, the C-14 disintigrates. Scientists determine the age of remains by comparing it's C-14 to the amount found in the living organism. The amount of C-14 present after t years is:

$$A(t) = A_0 e^{kt} \text{ w/ } k = -\left[\frac{\ln 2}{5600}\right]$$

ex// If $k = -0.5545$ for some substance, what is it's half-life?

Effective rate: If we invest \$1 at 8% interest (per year) compounded semi-annually we have $1(1.04)^2 = \$1.0816$ after 1 year. The actual amount of interest earned in that year was 8.16% rather then the 8% if it were to be compounded annually.

If r is the annual (stated) nominal rate of interest & m is the # of compounding periods:

$$\boxed{r_E = \left(1+\frac{r}{m}\right)^m - 1} \quad \text{(for compound interest)}$$

$$\boxed{r_E = e^r - 1} \quad \text{(for continuous compounding)}$$