

Formulas Continued . . .

Derivatives of Logs/Exponentials:

$$[a^{g(x)}]' = (a^{g(x)})(g'(x))(\ln a) \Rightarrow [a^x]' = (a^x)(\ln a)$$

$$[e^{g(x)}]' = (e^{g(x)})(g'(x)) \Rightarrow [e^x]' = e^x$$

$$[\log_a |g(x)|]' = \frac{g'(x)}{g(x) \ln a} \Rightarrow [\ln |g(x)|]' = \frac{g'(x)}{g(x)} \Rightarrow [\ln |x|]' = \frac{1}{x}$$

Curve Sketching:

1. Consider the domain of $f(x)$ and note any restrictions
2. x - intercept at $y = 0$, y - intercept at $x = 0$
3. Find asymptotes:
 - a) vertical if denominator = 0
 - b) horizontal if $\lim_{x \rightarrow \pm\infty} f(x)$ exists
4. Find critical points $x = c$ where $f'(x) = 0$ or $f'(x)$ d.n.e.
 - a) increasing where $f'(x) > 0$
 - b) decreasing where $f'(x) < 0$
5. Find relative extrema using part 4 or
 - a) $f'(c) > 0 \Rightarrow f(c)$ is a relative min at $x = c$
 - b) $f'(c) < 0 \Rightarrow f(c)$ is a relative max at $x = c$
6. Find inflection points where $f''(x) = 0$ or $f''(x)$ d.n.e.
 - a) concave up where $f''(x) > 0$
 - b) concave down where $f''(x) < 0$
7. Plot and connect all important points

Max/Min Problems:

1. Read the problem carefully, sketch if you can
2. Decide which variable (equation) to maximize or minimize, $f(x)$
3. Write this equation in terms of ONE variable
4. State the domain of $f(x)$ in terms of this variable
5. Find $f'(x)$, and the critical points and endpoints of $f(x)$
6. Test them all by plugging into $f(x)$
7. The absolute max is the largest of these values,
the absolute min is the smallest of these values

Area Formulas:

Area = (length)(width)

Volume = (length)(width)(height)

Area of a Triangle = $1/2$ (base)(height)

Area of a Circle = πr^2 (where r is the radius)

Circumference of a Circle = $2\pi r$

Antiderivatives:

$$1. \int kf(x)dx = k \int f(x)dx$$

$$2. \int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

$$3. \int x^n dx = \frac{x^{n+1}}{n+1} + C \quad \text{if } n \neq -1$$

$$4. \int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$$

$$5. \int e^x dx = e^x + C$$

$$6. \int e^{kx} dx = \frac{e^{kx}}{k} + C \quad \text{if } k \neq 0$$

$$7. \int a^x dx = \frac{a^x}{\ln a} + C$$

$$8. \int a^{kx} dx = \frac{a^{kx}}{k \ln a} + C \quad \text{if } k \neq 0$$

$$9. \int_a^b f(x)dx = F(b) - F(a) \quad \text{where } F(x) \text{ is the antiderivative of } f(x)$$

10. If the definite integral above represents area, it must be positive,

so find regions where $f(x) < 0$ and take $\left| \int_a^b f(x)dx \right|$ for those regions.

Multivariable Functions:

1. Partial fractions:

For $z = f(x, y)$:

$f_x =$ regard x as the variable, y as a constant

$f_y =$ regard y as the variable, x as a constant