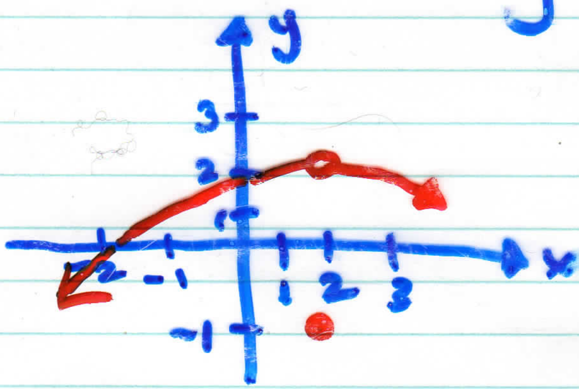
Section 3.2: Continuity

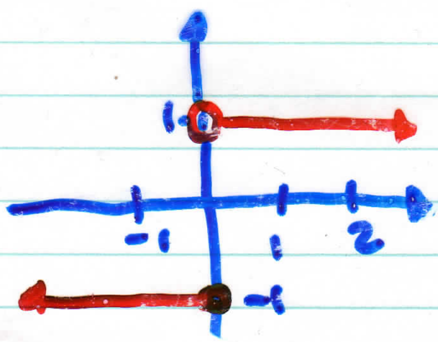
We have seen in the previous section that some curves are "smooth", some are undefined at certain spots, and some curves stop at one point & start at another. Curves that are smooth everywhere are called "continuous"

ex// Are the following graphs continuous?

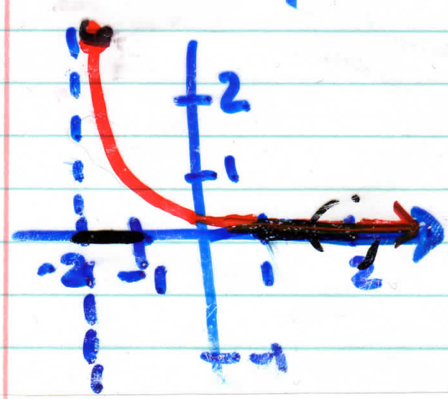
a)



b)



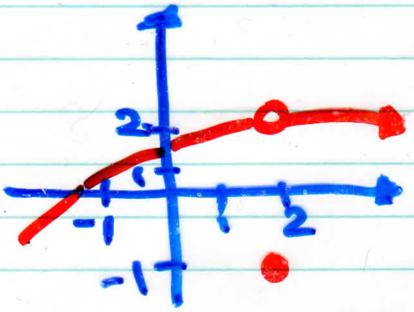
c)



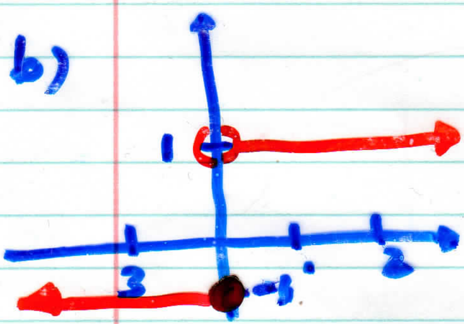
Defn': a fcn' $f(x)$ is "continuous" at a point $x=c$ if $\lim_{x \to c} f(x) = f(c)$.

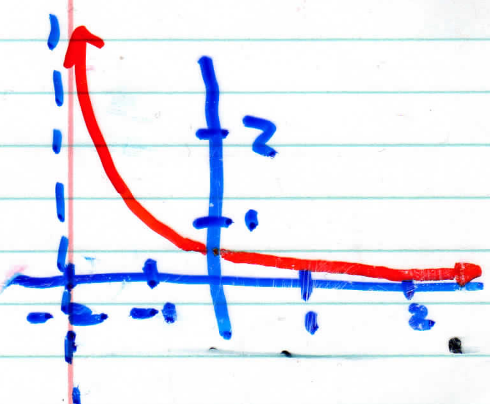
Lets look at our previous examples:

a)



b)





Section 3.3: Rates of Change

Suppose we know the distance ($f(t)$) that we have travelled at every moment as a function of time, t.

From this info, can we find a formula for the INSTANTANEOUS VELOCITY at every moment?

As $h \to 0$ (smaller & smaller time increments) we get "instantaneous velocity"

Defn: the "instantaneous rate of change" for a fcn' f when x=a is:

$$\boxed{\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

ex// Suppose the position of an object moving in a straight line is given by: $\boxed{s(t) = t^2 + 5t + 2}$

a) Find the average velocity from t=5 to t=1

Section 3.3: Rates of Change

Average Rate of Change: In some situations, we may want to know how one variable changes in relation to another.

The distance travelled during any time interval t_1 to t_2 can be calculated by $h(t_2)-h(t_1)$.

ex// Find the average speed over the interval $t=0$ to $t=3$.

Defn': The "average rate of change" of $f(x)$ w.r.t. x as x changes from a to b is:

$$\boxed{\frac{f(b)-f(a)}{b-a}}$$

ex// The percentage of men aged 65 & older in the workforce has been declining. The decline can be approximated by $f(x)=60(2)^x$ where x is the number of years since 1990. Find the average rate of change from 1992 to 1995.

Instantaneous Rate of Change: Sometimes, finding the average rate of change over very small intervals can be more helpful than finding the ave. rate of change over a large interval.

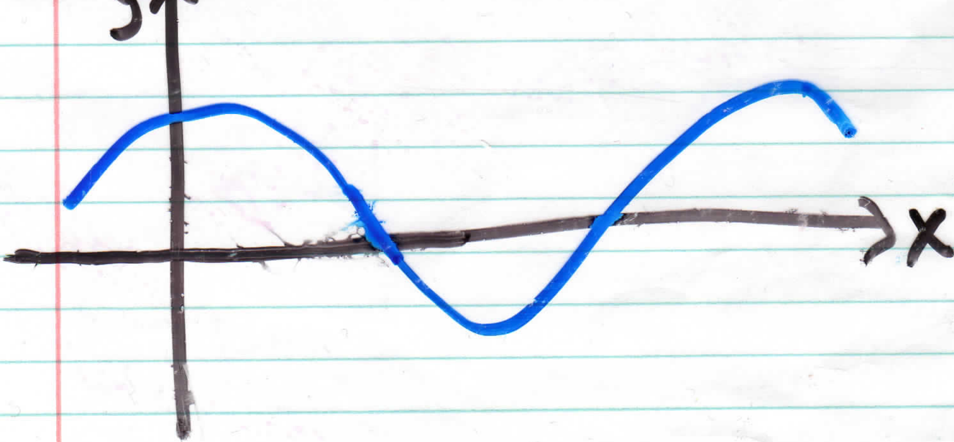
ex// A car starts & travels along a straight road, the distance travelled (in feet) in t seconds is given by $d(t) = t^2 + 3t$. After 10 seconds, the car has travelled $d(10) = 10^2 + 3(10) = 130$ feet. What is it's speed at 10 seconds?

Section 3.4: Derivative

In the previous section, $\boxed{\lim_{h \to} \frac{f(a+h)-f(a)}{h}}$ represented instantaneous rate change.

It can also have a geomet D meaning.

Tangent Line: For any curve $f(x)$, we can draw a "tangent line" at a point $x = P$



Secant Line:

as $h \rightarrow 0$ S gets closer to R & the secant line gets closer & closer to the tangent line at R. So the slope of the tangent line at a point $(a, f(a))$ is:

$$\boxed{\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

ex// Find the equation & of the tangent line to the curve $f(x) = 6 - x^2$, at $x = -1$.

The Derivative: The "derivative" of the function f at x is defined as $\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$

notice: the derivative is also a function of x, not a single number like the instantaneous rate of change or slope of the tangent line, both of which are just the derivative evaluated at a point, $f'(a)$.

ex// For f(x), find & Interpret $f'(a)$