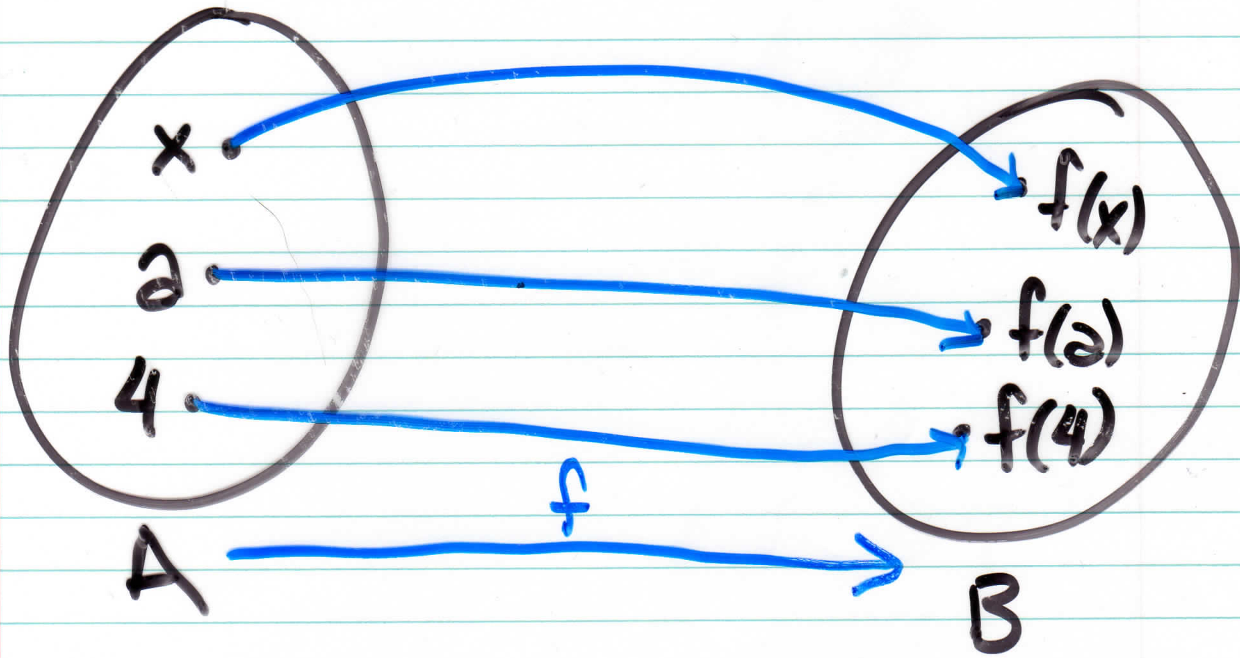
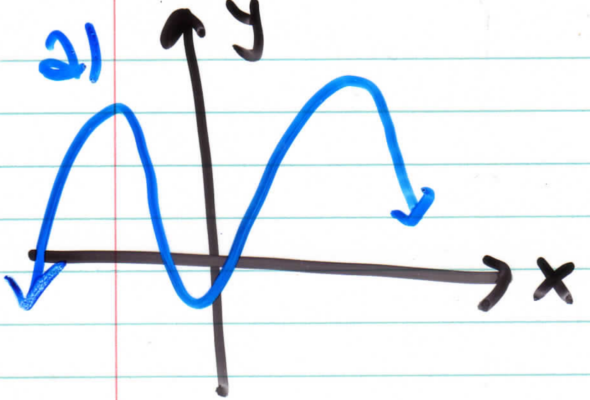
DAY 1: REVIEW OF FUNCTIONS

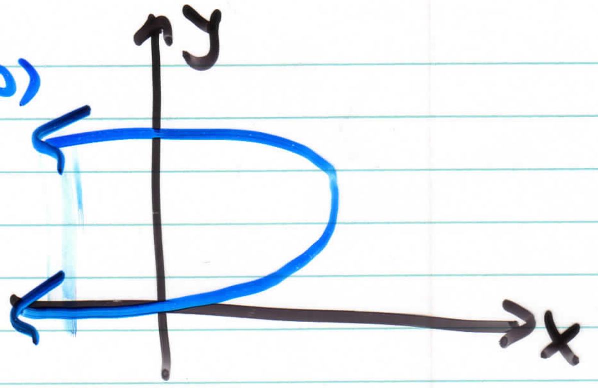
Defn': A "*function*" f is a rule that assigns to each element x in a set A, exactly one element $f(x)$ in a set B.

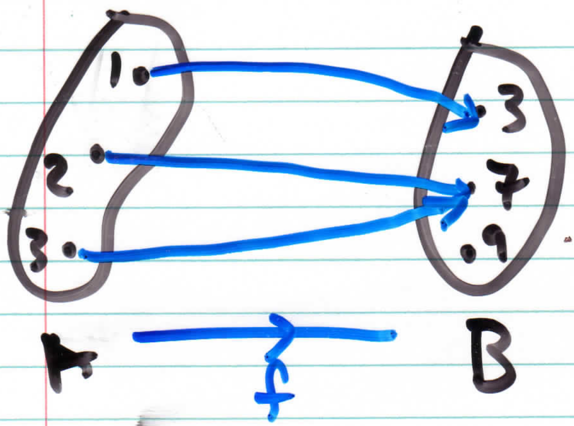


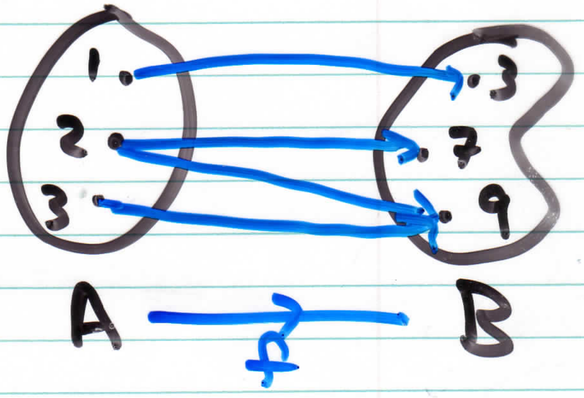
BUT: not all curves in the x-y plane are the graphs of functions (fcn's)!
A curve in the x-y plane is the graph of a fcn' of x if & only if no vertical line intersects the curve more than once - this is the "vertical line test".

ex// which of the following are fcn's?

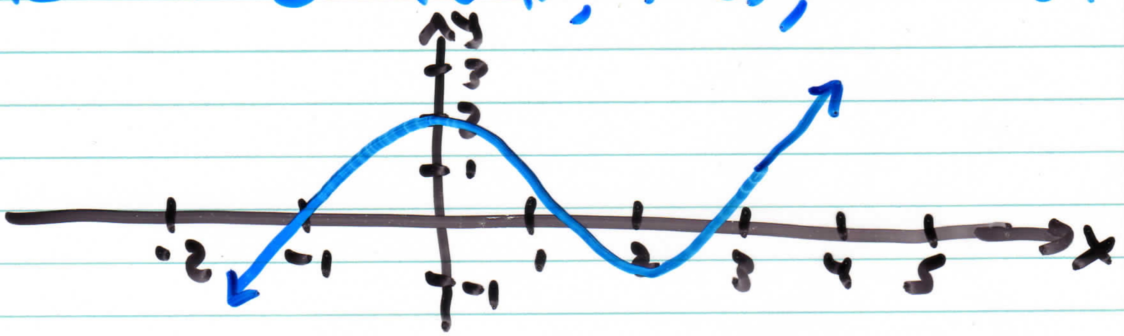
a) 

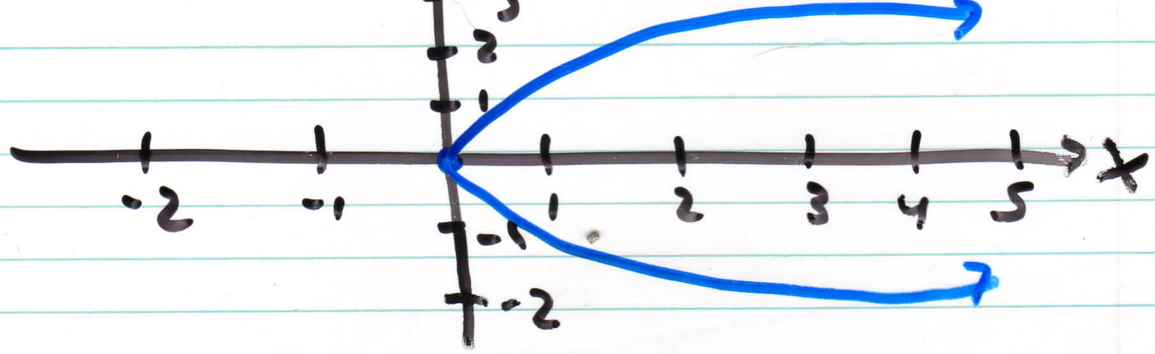
b) 

c) 

d) 

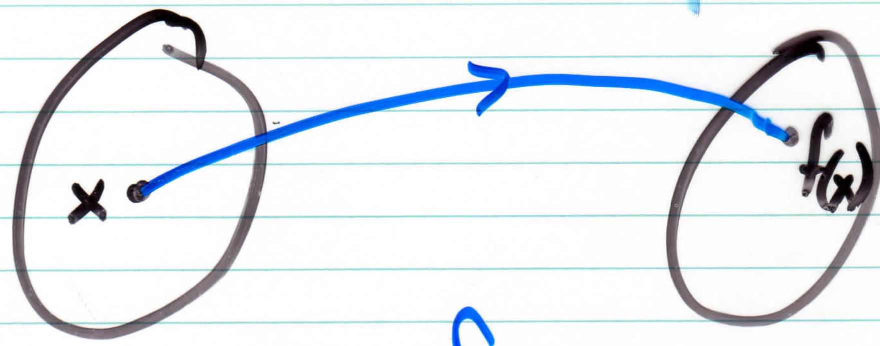
ex// use the graphs below to find the values of $f(-1)$, $f(0)$, & $f(3)$.

a) 

b) 

Features of a Function:

We usually consider functions for which the sets A & B are sets of real numbers.



$$A \xrightarrow{f} B$$

- The **DOMAIN** of f is the set of all possible values where f(x) is defined.
- The **RANGE** of f is all possible values of f(x) as x varies through the domain.
- The values in A are the *"independent variables"*.
- The values in B are the *"dependent variables"*.

Type of Fcn'	Domain
Polynomial Fcn' $p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$ ex// $f(x) = x^2$ $g(x) = x^3$	
Rational Fcn' $f(x) = \dfrac{p(x)}{q(x)}$ ex// $h(x) = \dfrac{x+1}{2x^2+x-1}$	
Root Fcn' $f(x) = \sqrt{p(x)}$ ex// $g(x) = \sqrt{x^2 - x}$	

Piecewise Defined Fcn's: Some functions are defined by different formulas in different parts of their domains.

ex// Find the domain & sketch a graph of

$$f(x) = \begin{cases} 2x+3 & \text{if } x < -1 \\ 3-x & \text{if } x \geq -1 \end{cases}$$

ex// Sketch the "absolute value function" - $|x|$.

Some basic fcn's :

$y = x$:

$y = x^2$:

$y = x^3$:

$y = \sqrt{x}$:

$(x-h)^2 + (y-k)^2 = r^2$:

<u>Symmetry</u>: There are 2 types of symmetries that we discuss when we talk about fcn's:

1) **EVEN** functions: these satisfy $f(-x)=f(x)$ & are symmetric about the y-axis.

ex// $f(x)=x^2$

2) **ODD** functions: these satisfy $f(-x)=-f(x)$. & are symmetric about the origin (180°).

ex// $f(x)=x^3$

ex// Are the following even, odd, or neither?

a) $f(x)=\dfrac{x^2}{x-1}$

b) $g(x)=\dfrac{x^2}{x^4-1}$

c) $h(x)=3x^3+x$

d) $p(x)=\dfrac{x^3}{-2x^5}$

New fcn's from old fcn' $y=f(x)$:

1. $y=f(x)+c \Rightarrow$ shift c units up $(c>0)$
2. $y=f(x)-c \Rightarrow$ shift c units down $(c>0)$
3. $y=f(x-c) \Rightarrow$ shift c units right $(c>0)$
4. $y=f(x+c) \Rightarrow$ shift c units left $(c>0)$
5. $y=cf(x) \Rightarrow$ stretches vertically by c $(c>1)$
6. $y=\frac{1}{c}f(x) \Rightarrow$ compress vertically by c $(c>1)$
7. $y=f(cx) \Rightarrow$ compress horizontally by c $(c>1)$
8. $y=f(\frac{1}{c}x) \Rightarrow$ stretch horizontally by c $(c>1)$
9. $y=-f(x) \Rightarrow$ reflect about the x-axis
10. $y=f(-x) \Rightarrow$ reflect about the y-axis.

The "exponential fcn" is a fcn' of the form:

$$f(x) = a^x$$

(It should not be confused with the power function x^n (ie x^2) where the variable is the base).

① $x^n = \underbrace{x \cdot x \cdot x \cdots x}_{n \text{ times}}$

② $x^{-n} = \frac{1}{x^n}$

③ $x^{1/n} = \sqrt[n]{x}$

④ $x^{m/n} = \left(\sqrt[n]{x}\right)^m = \sqrt[n]{x^m}$

e: of all possible bases for an exponential fcn', there is one that arises in many applications of calculus → $e \approx 2.71828$.

ex// Sketch the graphs of 2^x & $\left(\frac{1}{2}\right)^x$: