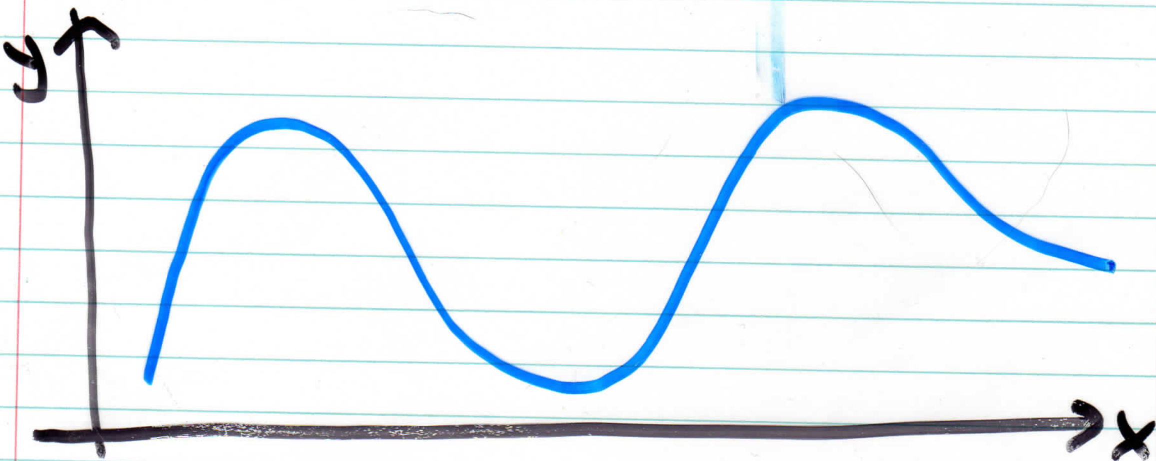
Chapter 4: APPLICATIONS OF DIFFERENTIATION

Section 4.2: Increasing/Decreasing Fcn's



This fcn' increases in some areas, & decreases in others. We can tell just from a fcn's eqn' where it increases/decreases

? How ?

Inc/Dec: if $f(x)$ has a derivative, then

1. if $f'(x)>0$ for each x in an interval; $f(x)$ is increasing.
2. if $f'(x)<0$ for each x in an interval; $f(x)$ is decreasing.
3. if $f'(x)=0$ for each x in an interval, $f(x)$ is constant.

? How do we find these intervals ?

The derivative changes signs (+ to −, or − to +) at the points where:

$f'(x)=0$ of $f'(x)$ d.n.e.

If these points are in the domain of $f(x)$, they are called "critical points."

Section 4.3: Relative Maxima & Minima

If c is a number in the domain of a fcn, then $f(c)$ is a "relative maximum" if there exists an open interval (a,b) containing c such that $f(x) \leq f(c)$ for all x in (a,b), & $f(c)$ is a "relative minimum" if $f(x) \geq f(c)$ for all x in (a,b).

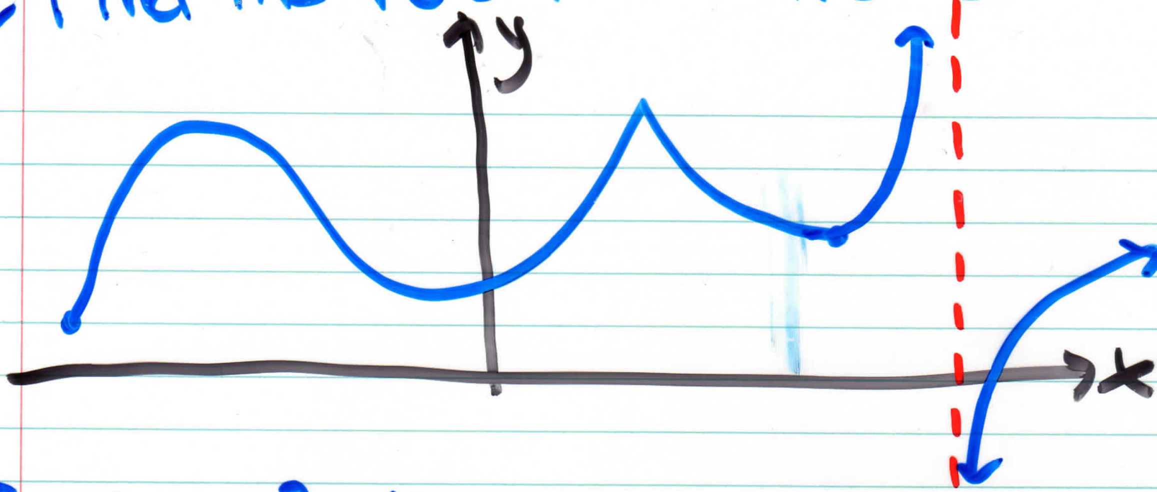
? How do we test if a critical point or endpoint is a relative max or min ?

1) $f(c)$ is a relative max of f if $f'(x)$ is positive on (a,c) & negative on (c,b).

2) $f(c)$ is a relative min of f if $f'(x)$ is negative on (a,c) & positive on (c,b).

ex// Find the relative extrema.

a)



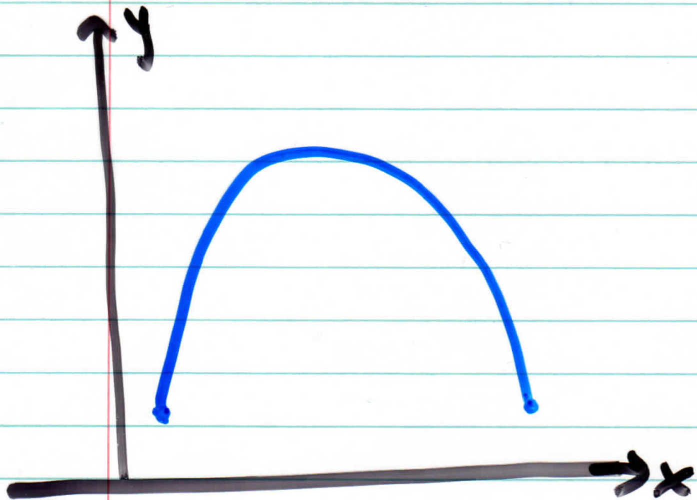
b) $f(x) = x^2 - 4x + 6$

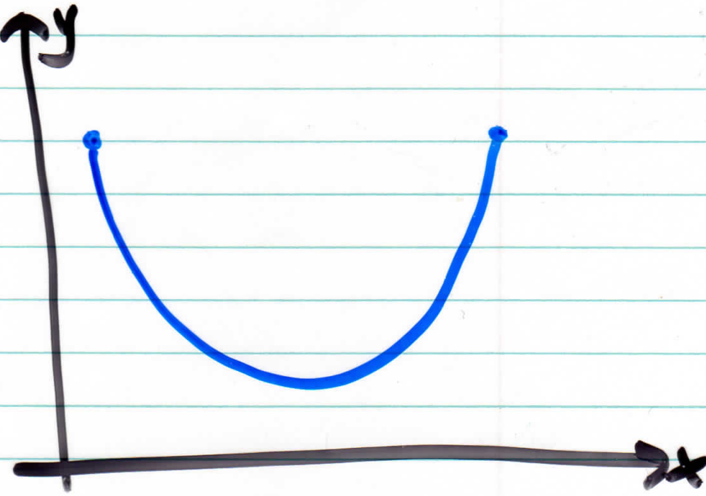
c) $f(x) = 3xe^x + 2$

Section 4.4: Concavity & Points of Inflection

We know that $f'(x)$ can tell us about a fcn's regions of inc/dec.
But, we may also be interested in <u>how</u> it is increasing/decreasing (ie, the rate at which inc/dec changes).

Look at the following 2 fcn's:

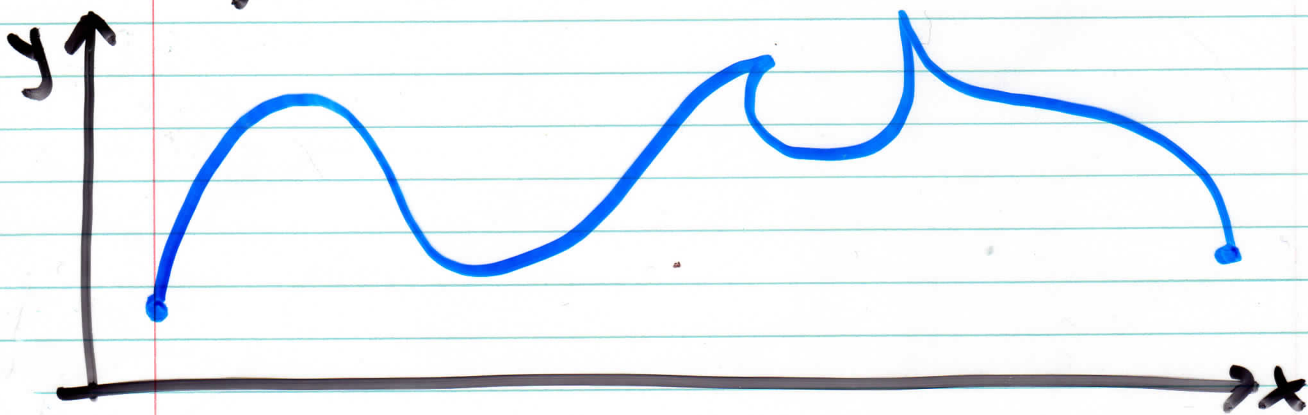




Concavity Test: for a fcn' $f(x)$ on interval I,

1) if $f''(x) > 0$ for all x in I, f is CU on I.
2) if $f''(x) < 0$ for all x in I, f is CD on I.

Where f changes from CU to CD (or CD to CU), we have an "*inflection point*".



ex// A particle's position fcn' is given by
$$s(t) = -x^3 + 66x^2 + 1050x - 400.$$
Use it's first & second derivatives to discuss it's motion.

The second derivative can also tell us if a critical point is a relative max/min.

1) if $f''(c) > 0$, $f(c)$ is a relative min.

2) if $f''(c) < 0$, $f(c)$ is a relative max.

3) if $f''(c) = 0$, no info about relative extrema.

Some possible shapes based on f' & f'':

f'' \ f'	$+,+$	$+,-$	$-,+$	$-,-$
$+$				
$-$				

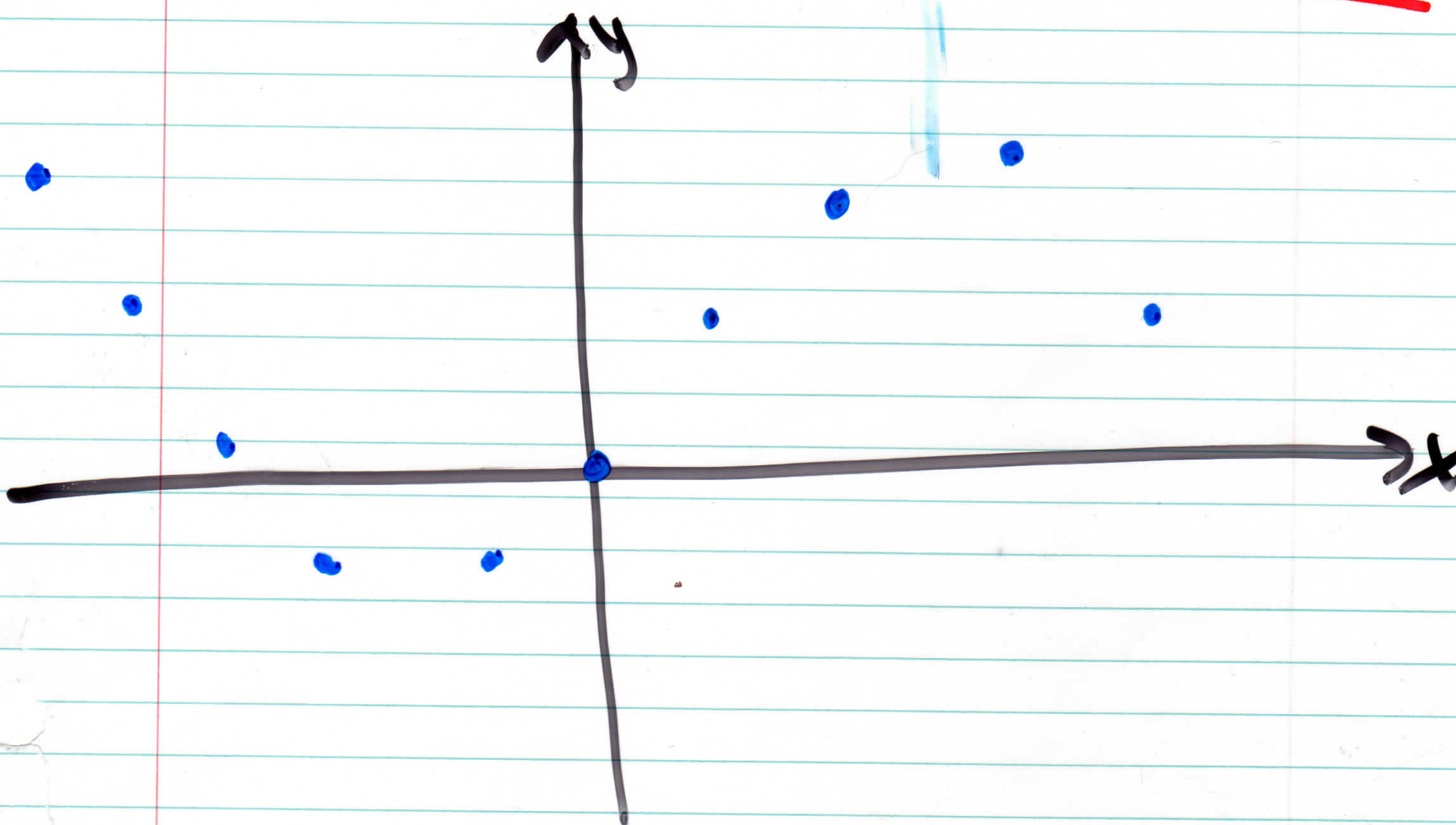
ex// sketch a possible graph of a fcn' that would satisfy the following:

i) $f'(x) > 0$ on $(-\infty, 1)$, $f'(x) < 0$ on $(1, \infty)$.

ii) $f''(x) > 0$ on $(-\infty, -2)$ & $(2, \infty)$, & $f''(x) < 0$ on $(-2, 2)$

iii) $\lim\limits_{x \to -\infty} f(x) = -2$ & $\lim\limits_{x \to \infty} f(x) = 0$.

Section 4.5 Drawing Graphs with Calculus



If we try to draw a graph by plotting points & connecting...

Guidlines For Curve Sketching

1. Domain: check where $f(x)$ is undefined.

2. Intercepts: y intercept → set $x=0$
   x intercept → set $y=0$

3. Symmetry (optional): if $\underbrace{f(-x)=f(x)}_{\text{even}}$ or $\underbrace{f(-x)=-f(x)}_{\text{odd}}$

4. Asymptotes: HA → check $\lim_{x\to\pm\infty} f(x)$ for $y=HA$
   VA → check $\lim_{x\to a} f(x)=\pm\infty$ for $x=VA$
   where $f(a)$ is undefined.

5. Inc/Dec: Compute f' & find the intervals where $f'(x)>0$ & $f'(x)<0$, & critical numbers where $f'(x)=0$ or $f'(x)$ d.n.e.

6. Local Max/Mins: Check critical numbers from 5 to see if they are max/mins.

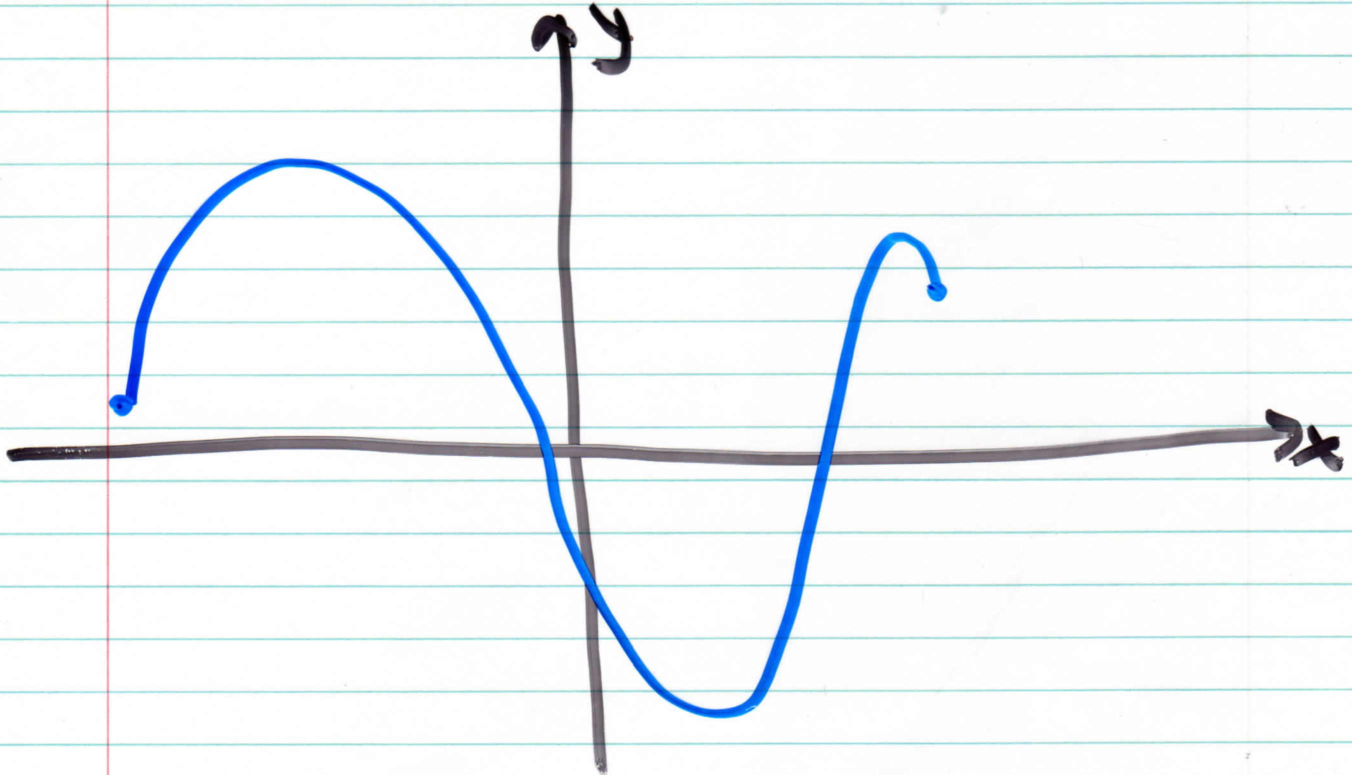
7. Concavity: compute f'' & find the possible inflection points. Check where $f''(x)>0$ & $f''(x)<0$ & confirm inflection points.

8. Label important points & sketch!

Section 4.7: Absolute Maxima & Minima

Defn': The "absolute (global) max" of a fcn' $f(x)$ on an interval I is $f(c)$ (at $x=c$) if $f(c) \geq f(x)$ FOR ALL x in I.

The "absolute (global) min" is $f(c)$ (at $x=c$) if $f(c) \leq f(x)$ FOR ALL x in I.



Extreme Value Thm': A fcn' that is cts on a closed interval I must attain an abs max & abs min.

Optimization Problems:

The methods we have learned in Ch 4 for finding extreme values have practical applications in many areas of life.
For example, minimizing cost, time, distance, or maximizing profit, area, volume, etc.

Steps:

1. Read carefully, what do I know?
   what do I need?
2. Draw a diagram.
3. Assign values to the variables involved.
4. Decide which variable is to be maximized/minimized & express it in terms of the others.
5. Use what you know to eliminate all but 1 variable in the eqn' from 4, call it *.
6. Write the domain of *.
7. Use the methods from this chapter to find the abs. max/min.

ex// A farmer has 2400 ft. of fencing & wants to fence off a rectangular field that borders a straight river. What are the dimensions of the field that has the largest area?

ex// A cylindrical can is to be made from 1000 mL of oil. Find the dimensions that will minimize the cost of the metal used to make the can.

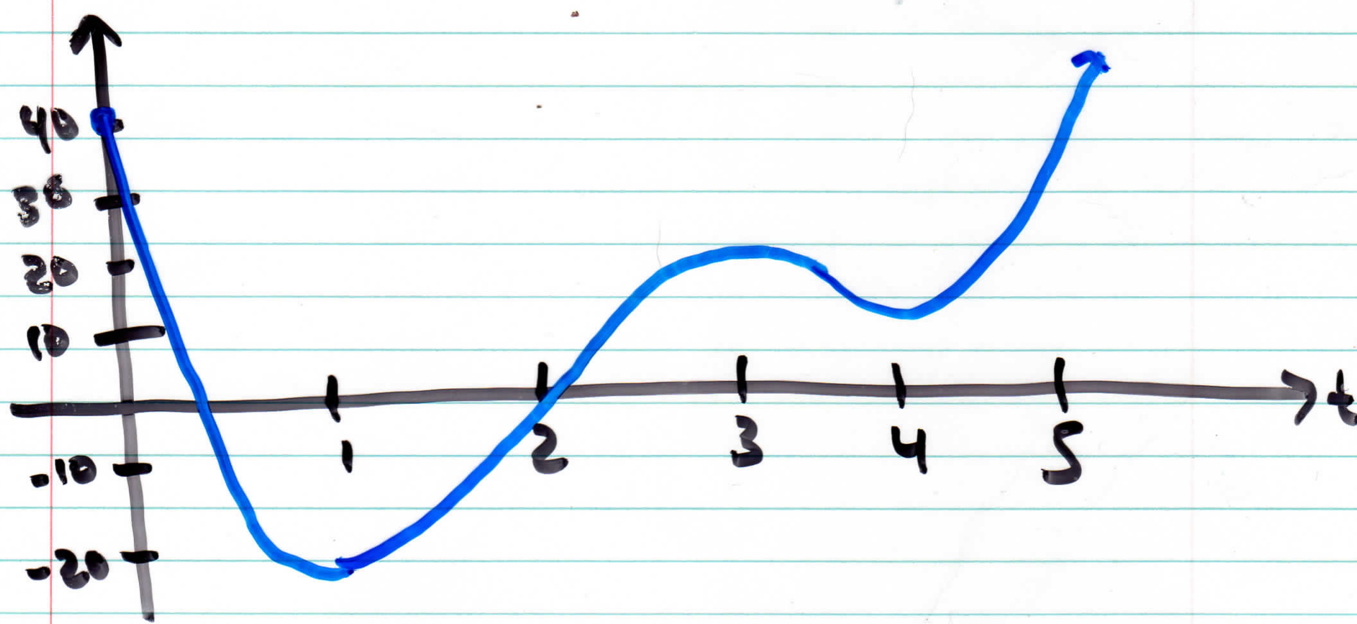
ex// A rectangular beam is to be cut from a circular log. Naturally, it is desirable for the beam to be as strong as possible, so it should be deeper than it is wide.

Experimental evidence in structural engineering has shown that the strength of a beam is proportional to the product of its width & the square of its depth. Find the dimensions of the strongest beam that can be cut from a circular log.

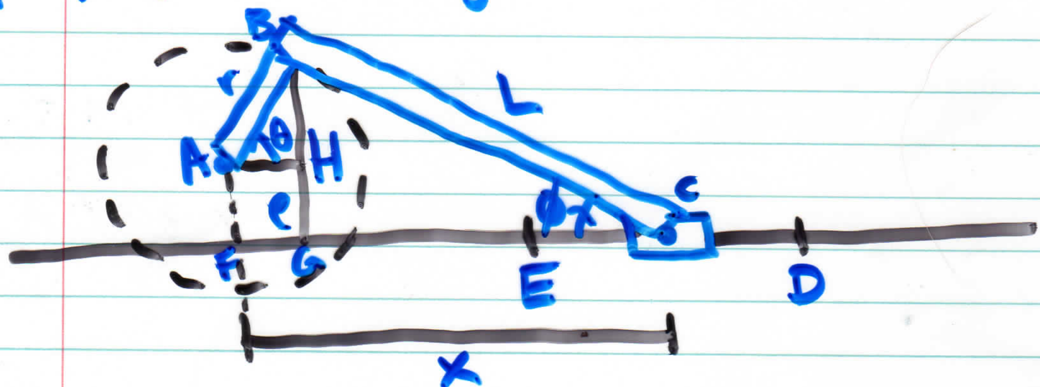
Section 4.8: Velocity & Acceleration.

The study of motion (relationships among displacement, velocity, & acceleration) is called "kinematics."

We have only gone deep enough to fully understand these ideas in 2D → objects moving in straight lines.



ex// Rod AB in the "offset slider-crank" below rotates counterclockwise with constant angular speed ω m/s about A. End C of the "follower" BC is confined to straight-line motion along a horizontal line b/w D & E. Find expressions for the velocity & acceleration of slider, C.



Section 3.10: Related Rates

In this section we see (word) problems where we need to compute the rate of change of one quantity in terms of the rate of change of another quantity. The process is to find an eqn' that relates the two (or more) quantities & then to differentiate both sides of the equation with respect to time.

ex// Air is being pumped into a spherical balloon, so that its volume increases at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

ex// A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft. away from the wall?

ex/ A water tank has the shape of an inverted circular cone with base radius 2m & height 4 m. If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 3m deep.

ex// Car A is traveling west at 50 mph & car B is traveling north at 60 mph. Both are headed for the intersection of the two roads. At what rate are the cars approaching each-other when car A is 0.3 miles & car B is 0.4 miles from the intersection?

ex/ A man walks along a straight path at a speed of 4 ft/s. A searchlight is located on the ground 20 ft. from the path & is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the light?

ex// One end of a rope is tied to a box. The other end is passed over a pulley 5m above the floor & tied at a level 1m above the floor to the back of the truck. If the rope is taut & the truck moves at $\frac{1}{2}$ m/s, how fast is the box rising when the truck is 3m from the plumbline through the pulley?