Chapter 5: THE INDEFINITE INTEGRAL & THE ANTIDERIVATIVE

Section 5.1: The Reverse Operation of Differentiation

Say we know the velocity of a particle but we want to know its position at a given time, or, we know the rate at which a population is growing, but we want to know its size at a given time...

In other words: given the derivative, can we work backwards to find the original?

A fcn' F is called an "antiderivative" of f on an interval I if $f(x) = F'(x)$ in I.

So, if F & G are any 2 antiderivatives of f, then $F'(x) = f(x) = G'(x)$ so $G(x) - F(x) = C$.

<u>Thm'</u>: If F is an antiderivative of f on an interval I, then the most general antiderivative f on I is: $F(x) + C$

By assigning specific values to C, we obtain a family of fcn's whose graphs are vertical translates of one another.

ex// Members of the family of antiderivatives of $f(x) = x^2$:

Table of Antiderivatives

We can use the "indefinite integral" to represent the antidifferentiation process. $\int$

$\int \sin x\,dx = -\cos x + C$

$\int \cos x\,dx = \sin x + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \sec x \tan x\,dx = \sec x + C$

$\int \csc^2 x\,dx = -\cot x + C$

$\int \csc x \cot x\,dx = -\csc x + C$

$\int e^x\,dx = e^x + C$

$\int a^x\,dx = \frac{a^x}{\ln a} + C$

$\int \frac{1}{x}\,dx = \ln|x| + C$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

Antiderivative Laws:

$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

$\int k f(x)\,dx = k \int f(x)\,dx \quad$ (k a constant)

In applications of calculus, it is very common to have a situation where it is required to find a fcn' given knowledge about its derivative. An eqn' that involves the derivatives of some fcn' is called a "differential equation".

Can we ever determine C uniquely?
There may be some extra conditions given that will uniquely specify the soln'.

ex// find $f(x)$ if $f'(x) = e^x + 20(1+x^2)$

We can also find the antiderivative multiple times (as we could find f', f'', f''', etc.)

ex// Find $f(x)$ if $f''(x) = 12x^2 + 6x - 4$ &

a) $f(0) = 4$ & $f(1) = 1$

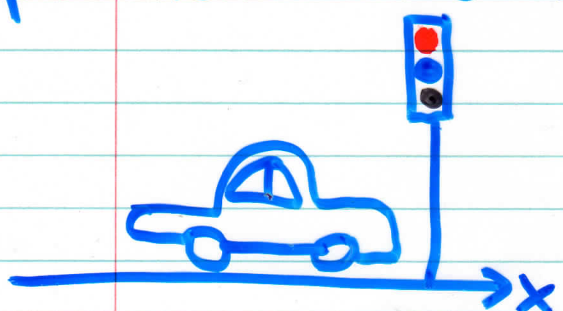
b) $f(0) = 7$ & $f'(0) = 0$

Notice: when we started with f'' we needed 2 pieces of info because we had 2 unknown constants. In fact, when finding f from $f^{(n)}$, we will end up with n unknown constants, & therefore we will need n conditons to find a unique solution.

Section 5.2: Integrating Velocity & Acceleration

Rectilinear Motion: Recall that if an object has position fcn' $s = f(t) \Rightarrow$ velocity $v(t) = s'(t)$. So position is the antiderivative of velocity! Likewise, acceleration $a(t) = v'(t) = s''(t)$, so velocity is the antiderivative of acceleration!

ex// A car accelerates from rest when the light turns green. Initially, the acceleration is $10\ m/s^2$ but it decreases linearly, reaching 0 after 10s. Find the velocity & position of the car during this interval.



Section 5.3: Change of Variable in the Indefinite Integral

Sometimes we can adjust what we must integrate by introducing a change of variables. This is sometimes known as ,
and it is most effective when both a fcn & its derivative appear in our integral.

ex// Evaluate $\int 2x\sqrt{x^2+1}\,dx$

Thm: Suppose the substitution $u=h(x) \Leftrightarrow x=g(u)$ makes $\int f(x)dx = \int f(g(u))\cdot g'(u)du$.
Then if $F(u)$ is an antiderivative of $f(g(u))\cdot g'(u)$

$$\Rightarrow \int f(x)dx = F(h(x)) + C.$$