Topic 6 Outline

Topic 6 Learning Objectives

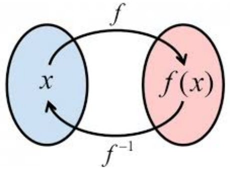
1. define an inverse function
2. understand the term "one-to-one" and the horizontal line test
3. understand and use the relationship between exponentials and logarithms
4. define and graph the logarithm function
5. recall basic facts about the logarithmic function
6. recall and use the logarithm laws (ie, to solve equations)
7. define and graph the natural logarithm
8. recall basic facts about the natural logarithm
9. differentiate logarithmic and exponential functions using the rules
10. recognize when logarithmic differentiation is useful or necessary
11. use logarithmic differentiation to differentiate functions

Inverse Functions

Lets say that we observe a population of bacteria. The size of the bacterua is recorded hourly, so that the number of bacteria N can be thought of as a function of time t: $N = f(t)$.

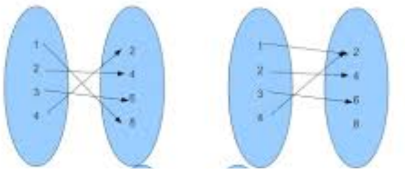
Say instead that we want to study the tume required for the population to reach various levels. We are now thinking of t as a function of N: $t = g(N)$.

This is the **inverse** function of f, denoted $f^{-1}(N)$.



Inverse Functions

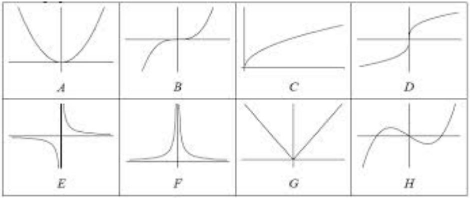
Not all functions have inverses:



f is called **one-to-one** because it never takes on the same output twice.

Inverse Functions

To distinguish whether something was a function or not, we used the vertical line test. To test if a function is one-to-one (and thus has an inverse), we can use the **horizontal line test**!
Why do you think this works?? What does it tell you about the relationship between x- and y-coordinates of a function and its inverse?
Example: Are the following functions 1-1?

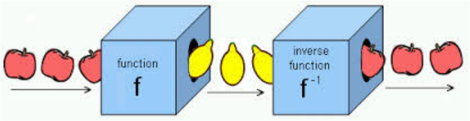
1. 

2. $f(x) = x^2$?
3. $f(x) = x^3$?
4. $f(x) = a^x$ for values of $a > 0$?

Inverse Functions

So the domain of $f(x)$ is the range of $f^{-1}(x)$, and vice versa!

Cancellation Equations: $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$



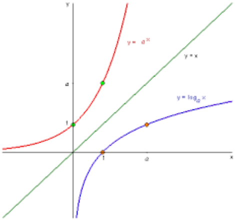
Examples

1. Find the inverse of of the function $f(x) = x^3 + 2$

2. Sketch the function $f(x) = \sqrt{-1 - x}$ and its inverse.

Logarithmic Functions

If $a > 0$ and $a \neq 1$, then the function a^x is 1-1 and has an inverse called the **logarithmic function with base a**, $log_a(x)$.



Domain: Range:

We can transform back and forth between logarithmic and exponential functions by using the relationship:

$$log_a x = y \Leftrightarrow a^y = x$$

Logarithmic Functions

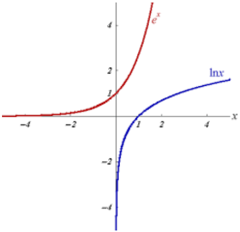
By the cancellation equations:

Log Laws:

1. $log_a(xy) = log_a x + log_a y$
2. $log_a(\frac{x}{y}) = log_a x - log_a y$
3. $log_a(x^r) = r log_a x$

The Natural Logarithm

Of all possible bases for *log*, the base *e* is the most frequently used. We call this the **natural logarithm**, $log_e x = ln x$.



Facts to remember:

Examples

1. Evaluate $log_2(80) - log_2(5) = x$
2. Solve for x in $logx = 5$
3. Solve for x in $e^{5-3x} = 10$
4. Solve for x in $2ln(4x) = 1$
5. Express $ln(1 + x^2) + \frac{1}{2}lnx - ln(sinx)$ as a single logarithm
6. Find the domain of $f(x) = log(3 - x)$
7. Find the domain of $f(x) = \sqrt{3 - e^{2x}}$
8. Find the domain of $f(x) = ln(2 + lnx)$

Derivatives of Logs and Exponentials

1. $\frac{d}{dx}(log_a g(x)) = \frac{g'(x)}{g(x) lna}$
2. $\frac{d}{dx}(log_a x) = \frac{1}{x lna}$
3. $\frac{d}{dx}(lng(x)) = \frac{g'(x)}{g(x)}$
4. $\frac{d}{dx}(lnx) = \frac{1}{x}$
5. $\frac{d}{dx}(a^{g(x)}) = a^{g(x)} g'(x) lna$
6. $\frac{d}{dx}(a^x) = a^x lna$
7. $\frac{d}{dx}(e^{g(x)}) = e^{g(x)} g'(x)$
8. $\frac{d}{dx}(e^x) = e^x$

Check out this link for a video on the log functions and their derivatives!
https://www.educreations.com/lesson/embed/9773346/?ref=app

Examples

Find $f'(x)$ if $f(x) =$

1. $ln(sinx)$

2. $10^{x^2} + 3^x$

3. $log_{10}(\frac{x}{x-1})$

4. $\sqrt{lnx}$

5. $ln|x|$

Logarithmic Differentiation

Sometimes, the calculation of derivaties of complex functions can be made easier by taking logarithms! Steps:

1. Take natural logs (or any other base would also do) of both sides of an equation that is in the form $y = f(x)$ and use the log laws to simplify it.
2. Use implicit differentiation with respect to x to differentiate.
3. Solve for y', and this is the derivative we were looking for!

Example: Differentiate $y = \frac{x^{\frac{3}{4}}\sqrt{x^2+1}}{(3x+2)^5}$

Logarithmic Differentiation

In other cases, Logarithmic differentiation is necessary because none of the other rules will do!
This happens in cases where our function has the form $y = f(x)^{g(x)}$.
Example: Differentiate $y = x^{sinx}$

Five in Five!

1. What is the domain of the function $ln(\frac{x}{x-1})$?

2. Sketch a graph of the function $log_2 x$.

3. Differentiate $y = ln(e^x x^3)$.

4. Differentiate $y = 3^x + x^3 + log_3 x$.

5. Differentiate $y = x^x$

Flex the Mental Muscle!

1. Differentiate $y = x^{cos3x} + 7^{x^2}$, $x > 0$.

2. Differentiate
$$y = \frac{\sqrt[3]{x - tanx}(1 + 2x^3)^5}{\sqrt{1 + x^2}}$$
once without logarithmic differentiation, and once with. Simplify your final answers until they match.