Math 1500

Course Notes

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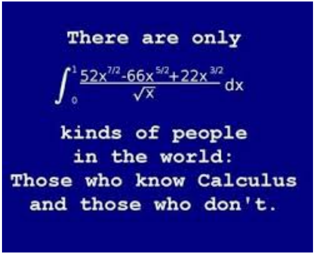


Topic 1 Outline

1. Functions and Models
   - Four Ways to Represent a Function
   - New Functions from Old Functions
   - The Exponential Function

Topic 1 Learning Objectives

1. define a function visually, numerically, and algebraically
2. sketch basic functions
3. find domain and range for various functions
4. sketch and describe piecewise defined functions
5. describe some basic features of functions
6. sketch and describe modifications of basic functions
7. describe combinations and compositions of functions
8. define and describe the exponential function
9. graph exponential functions
10. define base e for the exponential function

Four Ways to Describe a Function

A **function** is the fundamental object that we deal with in Calculus.
Functions arrise whenever one quantity depends on another.
Check out this link for a video on functions!
https://www.educreations.com/lesson/embed/9666672/?ref=app
There are four ways to describe a function:

1. verbally
2. numerically
3. visually
4. algebraically

Definition

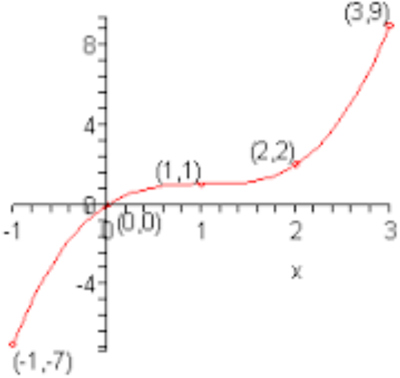
A **function** f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B.

Example

Find the values of $f(-1)$, $f(0)$, and $f(2)$ if $f(x) = x^2 - 2x + 1$.

Example

Using the graph below, find the values of $f(-1)$, $f(0)$, and $f(3)$.

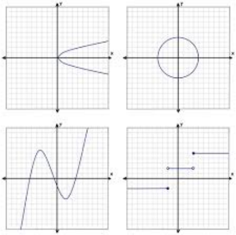


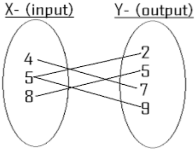
Note

Not all curves in the $x - y$ plane are the graphs of functions! A curve in the $x - y$ plane is the graph of a function $f(x)$ if and only if no vertical line intersects the curve more than once. This is the *Vertical Line Test*
Why do you think this works??

Example

Which of the following are functions?





1.

2. $y = x^2$

3. $x^2 + y^2 = a^2$

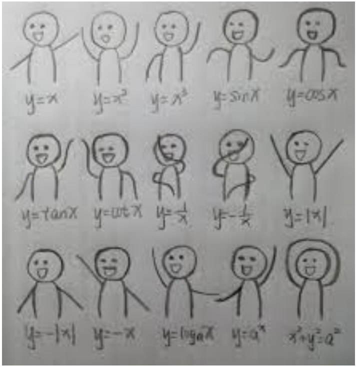
Basic Fuctions

Graph the following basic functions

1. $y = ax$
2. $y = x^2$
3. $y = x^3$
4. $y = |x|$
5. $y = \sqrt{x}$
6. $y = \frac{1}{x}$

Basic Functions

Here's a nice way to remember your basic functions!



From https://mathematicianincognito.wordpress.com

Features of a Function

We will be considering functions for which the set of inputs and set of outputs are real numbers.

- The **DOMAIN** of a function f is the set of all possible values for which $f(x)$ is defined.
- The **RANGE** of a function f is the set of all possible values of $f(x)$ as x varies through the domain.
  - The numbers/symbols in the domain are called the "independent variables".
  - The numbers/symbols in the range are called the "dependent variables".

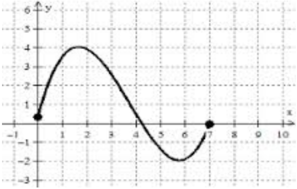
Features of a Function

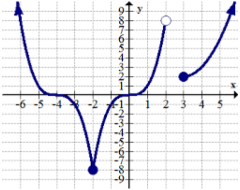
Type	General	Domain	Example	Domain
Polynomial				
Rational				
Root				

Table : Types of Functions and their Domains

Example

State the domain and range (for 1-3) for the following functions:

1. 



X	Y
2	3
1	4
0	5
-1	6
-2	7

2. $f(x) = x^2$

3. $g(x) = \frac{3}{x^2-2x}$

Piecewise Defined Functions

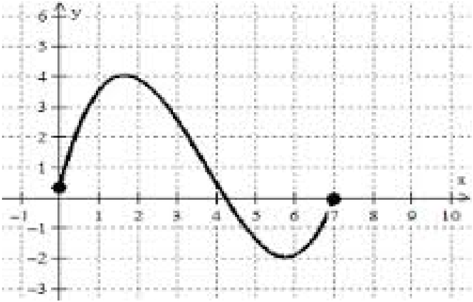
Piecewise defined functions are defined by different functions over different parts of their domain. Sketch and find the domain for the following piecewise defined functions:

- $$f(x) = \begin{cases} 2x+3 & \text{if } x < -1 \\ -x^2 & \text{if } x \geq -1 \end{cases}$$

- $f(x) = |x|$

Increasing/Decreasing

A function $f(x)$ is **increasing** on an interval I if $f(x_1) < f(x_2)$ for $x_1 < x_2$ in I. It is **decreasing** on an interval I if $f(x_1) > f(x_2)$ for $x_1 > x_2$ in I.



Symmetry

There are 2 types of symmetries that we discuss when we talk about functions:

- **EVEN** functions satisfy $f(-x) = f(x)$, and are symmetric about the y-axis.
  - ex: $f(x) = x^2$
- **ODD** functions satisfy $f(-x) = -f(x)$, and are symmetric about the origin.
  - ex: $f(x) = x^3$

New Functions from Old Functions

Once we know our basic functions, we can quickly sketch graphs and write equations for related functions by following some simple rules. If we know $y = f(x)$:

1. $y = f(x) + c \Rightarrow$ shift c units up $(c > 0)$.
2. $y = f(x) - c \Rightarrow$ shift c units down $(c > 0)$.
3. $y = f(x + c) \Rightarrow$ shift c units right $(c > 0)$.
4. $y = f(x - c) \Rightarrow$ shift c units left $(c > 0)$.
5. $y = cf(x) \Rightarrow$ stretch vertically by c $(c > 1)$.
6. $y = \frac{1}{c}f(x) \Rightarrow$ compress vertically by c $(c > 1)$.
7. $y = f(cx) \Rightarrow$ compress horizontally by c $(c > 1)$.
8. $y = f(\frac{1}{c}x) \Rightarrow$ stretch horizontally by c $(c > 1)$.
9. $y = -f(x) \Rightarrow$ reflect about the x-axis.
10. $y = f(-x) \Rightarrow$ reflect about the y-axis.

New Functions from Old Functions

We can **combine** two functions $f(x)$ and $g(x)$ to form 4 new functions:

1. $f(x) + g(x)$ or $(f + g)(x)$
2. $f(x) - g(x)$ or $(f - g)(x)$
3. $f(x)g(x)$ or $(fg)(x)$
4. $\frac{f(x)}{g(x)}$ or $(\frac{f}{g})(x)$

If the domain of f was A and the domain of g was B, then the domain of any of these new functions is the intersection of A an B.

New Functions from Old Functions

We can **compose** two functions $f(x)$ and $g(x)$ by putting one of the *inside* the other one. The notation looks like $(f \circ g)(x)$ or $f(g(x))$.

Ex: If $f(x) = x^2$ and $g(x) = \sqrt{2 - x}$, find $f(g(x))$, $g(f(x))$, $f(f(x))$, and $f(g(f(x)))$

The Exponential Function

The **exponential function** is a function of the form:

$$f(x) = a^x$$

where a is a constant that is > 0, and x is a variable.
Check out this link for a video on exponential functions!
https://www.educreations.com/lesson/embed/9669753/?ref=app
To work with these functions, we must recall our rules for exponents:

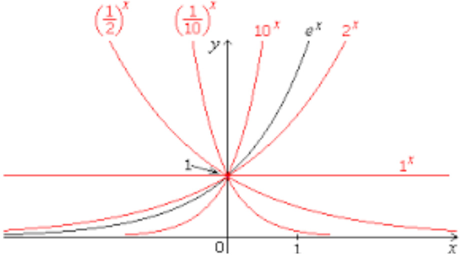
1. $a^n =$
2. $a^0 =$
3. $a^{-n} =$
4. $a^{\frac{n}{m}} =$

Example

Sketch the graphs of 2^x and $\left(\frac{1}{2}\right)^x$, and then estimate the value of $2^{\sqrt{3}}$.

The Exponential Function

Below are the graphs of some exponential functions:



Of all possible bases for an exponential function, there is one that is most convenient for calculus purposes. We call this number e, and $e \approx 2.71828...$

Five in Five!

Solve the following in 5 minutes or less!

1. Find the domain of $\frac{\sqrt{x}}{4-x}$

2. Are the following odd, even, or neither?
$f(x) = \frac{3}{x^3+x}$, $g(x) = x^4 - 4x^2$, $h(x) = 3x^3 + 2x^2 + 1$

3. For $f(x) = \frac{1}{x}$, $g(x) = 2x - 3$, and $h(x) = \sqrt{5x}$, find $f(g(h(x)))$

4. State the value of $9^{\frac{3}{2}}$

5. Sketch the piecewise defined function
$$f(x) = \begin{cases} x^3 & \text{if } x \leq 0 \\ e^x & \text{if } x > 0 \end{cases}$$

Flex the Mental Muscle!

The exponential function occurs frequently in mathematical models of nature and society, in particular, in the descriptions of population growth and decay.

The *half-life* of stronium-90 is 25 years. This means that half of any given quantity of stronium-90 will disintegrate in 90 years.

1. If a sample of stronium-90 has a mass of 24mg, find an expression for the mass $m(t)$ that remains after t years.

2. Find the mass remaining after 40 years.