DATE: February 29, 2008 | MIDTERM

DEPARTMENT & COURSE NO: MATH 1500 | TIME: 1 hour
EXAMINATION: Intro Calculus | EXAMINER: Various

1. Evaluate the following limits. If the limit does not exist or is $\pm\infty$ indicate that.

[4] (a) $\displaystyle\lim_{x\to 2} \frac{\sqrt{x+2}-\sqrt{2x}}{x-2}$

Solution:

$$\lim_{x\to 2} \frac{\sqrt{x+2}-\sqrt{2x}}{x-2} = \lim_{x\to 2} \frac{\sqrt{x+2}-\sqrt{2x}}{x-2}\cdot\frac{\sqrt{x+2}+\sqrt{2x}}{\sqrt{x+2}+\sqrt{2x}} \qquad 1$$

$$= \lim_{x\to 2} \frac{x+2-2x}{x-2}\cdot\frac{1}{\sqrt{x+2}+\sqrt{2x}} \qquad 1$$

$$= -\lim_{x\to 2} \frac{1}{\sqrt{x+2}+\sqrt{2x}} \qquad 1$$

$$= -\frac{1}{4} \qquad 1$$

BAD MATH
-1 for first

~~Colt Jordan~~

[3] (b) $\displaystyle\lim_{x\to -\infty} \frac{\sqrt{x^2+2x}}{2x+1}$

Solution:

$$\lim_{x\to -\infty} \frac{\sqrt{x^2+2x}}{2x+1} = -\frac{1}{2} \qquad 1+2$$

[4] (c) $\displaystyle\lim_{x\to(-2)^-} \frac{x^2+5x+6}{x(x+2)^2}$

Solution:

$$\lim_{x\to(-2)^-} \frac{x^2+5x+6}{x(x+2)^2} = \lim_{x\to(-2)^-} \frac{x+3}{x(x+2)} \qquad 1$$

$$= +\infty \qquad 1+2$$

NO WORK
→ NO MARK

-1 for d.n.e.

$-1/8 \quad 0$

$-4 \quad -3 \quad -2$

$\dfrac{-1}{-4(-2)}$

2. Find the derivative $f'(x)$ in each case. DO NOT SIMPLIFY your answers.

[4] (a) $f(x) = (5x)^5 + \dfrac{1}{5x} + (5x)^{1/5} + (5\pi)^{-5}$

Solution:

$5(5x)^4 \cdot 5$

-1 for no chain

$$f'(x) = 25(5x)^4 \quad 1$$
$$-\frac{1}{5x^2} \quad 1$$
$$(5x)^{-4/5} \quad 1$$
$$0 \quad 1$$

or $\frac{1}{5}(5x)^{-4/5} \cdot 5$

[4] (b) $f(x) = (x^2+1)\tan x$

Solution:

$$f'(x) = 2x\tan x \quad 1$$
$$+ \quad 2$$
$$(x^2+1)\sec^2 x \quad 1$$

[4] (c) $f(x) = \dfrac{1+\cos x}{1+\sin x}$

Solution:

$$f'(x) = (1+\sin x)(-\sin x) - (1+\cos x)\cos x \quad 2$$
$$\text{——————} \quad 1$$
$$(1+\sin x)^2 \quad 1$$

[4] (d) $f(x) = e^{\sqrt{x^2+1}}$

Solution:

$$f'(x) = e^{\sqrt{x^2+1}} \quad 1$$
$$\frac{1}{2\sqrt{x^2+1}} \quad 2$$
$$2x \quad 1$$

[6] 3. Let

$$f(x) = \begin{cases} x^2 + x & \text{if } x \geq -1; \\ k^2 + kx & \text{if } x < -1. \end{cases}$$

Find the value or values of k for which f is continuous at $x = -1$. You MUST use limits to justify your answers.

Solution:

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} k^2 + kx = k^2 - k$$ 1

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} x^2 + x = 0$$ 1

Therefore $\lim_{x \to -1} f(x)$ exists if $k^2 - k = 0$ or $k = 0, 1$ 2

If $k = 0$, then $\lim_{x \to -1} f(x) = 0 = f(-1)$ and therefore f is continuous at $x = -1$ 1

If $k = 1$, then $\lim_{x \to -1} f(x) = 0 = f(-1)$ and therefore f is continuous at $x = -1$. 1

[5] 4. If f is differentiable, prove that

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x).$$

NO LIM (-1)

Solution:

1 for defn

$$(cf)'(x) = \lim_{h \to 0} \frac{(cf)(x+h) - (cf)(x)}{h}$$ 2

$$= \lim_{h \to 0} \frac{cf(x+h) - cf(x)}{h}$$ 1

$$= c \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ 1

$$= cf'(x)$$ 1

Product Rule

(1)

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[8] 5. Let y be a function of x which satisfies the equation $2x + y - \sqrt{2}\sin(xy) = \dfrac{\pi}{2}$.
Find the slope of the tangent to the function at the point $\left(\dfrac{\pi}{4}, 1\right)$.

Solution:

$$2 + y' - [\sqrt{2}\cos(xy)](y + xy') = 0 \qquad 5$$

$$\text{at } (\pi/4, 1), \quad 2 + y' - (1 + \frac{\pi}{4}y') = 0 \qquad 2$$

$$y' = \frac{1}{\pi/4 - 1} \qquad 1$$

[6] 6. Use only the definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{1}{2x+1}$.

Solution:

for defn

NO LIM -1

$$f'(x) = \lim_{h\to 0} \frac{\dfrac{1}{2(x+h)+1} - \dfrac{1}{2x+1}}{h} \qquad 1+2$$

$$= \lim_{h\to 0} \frac{2x+1-(2(x+h)+1)}{h(2(x+h)+1)(2x+1)} \qquad 1$$

$$= \lim_{h\to 0} \frac{-2h}{h(2(x+h)+1)(2x+1)}$$

$$= \lim_{h\to 0} \frac{-2}{(2(x+h)+1)(2x+1)} \qquad 1$$

$$= \frac{-2}{(2x+1)^2} \qquad 1$$

7. A light sits on the ground 20m from a building. A man 2m tall walks away from the light directly toward the building at 1m/s. How fast is the length of his shadow on the building changing when he is 14m from the building?

Solution:

diagram

variables: distance from light$=x$, height of shadow $=s$ — 1

$$\frac{2}{x} = \frac{s}{20}$$ 1

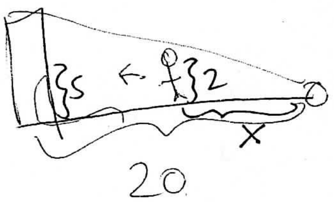
$$s = \frac{40}{x}$$ 1

$$\frac{dx}{dt} = 1; \quad \left.\frac{ds}{dt}\right|_{x=6} = ?$$ 1

$$\frac{ds}{dt} = -\frac{40}{x^2}\frac{dx}{dt}$$ 2

$$\left.\frac{ds}{dt}\right|_{x=6} = -\frac{40}{36}$$ or $-10/9$ m/s 1

The length of the shadow is decreasing at 10/9 m/sec. 1



$\frac{ds}{dt} = ?$

$\frac{dx}{dt} = 1$

$\frac{2}{x} = \frac{s}{20}$