UNIVERSITY OF MANITOBA

DATE: February 29, 2008

MIDTERM
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TIME: 1 hour
EXAMINER: Various

DEPARTMENT & COURSE NO: MATH 1500
EXAMINATION: Intro Calculus

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1. Evaluate the following limits. If the limit does not exist or is $\pm\infty$ indicate that.

[4] (a) $\lim_{x\to 2} \dfrac{\sqrt{x+2}-\sqrt{2x}}{x-2}$

[3] (b) $\lim_{x\to-\infty} \dfrac{\sqrt{x^2+2x}}{2x+1}$

[4] (c) $\lim_{x\to(-2)^-} \dfrac{x^2+5x+6}{x(x+2)^2}$

2. Find the derivative $f'(x)$ in each case. DO NOT SIMPLIFY your answers.

[4] (a) $f(x) = (5x)^5 + \dfrac{1}{5x} + (5x)^{1/5} + (5\pi)^{-5}$

[4] (b) $f(x) = (x^2 + 1)\tan x$

[4] (c) $f(x) = \dfrac{1 + \cos x}{1 + \sin x}$

[4] (d) $f(x) = e^{\sqrt{x^2+1}}$

[6] 3. Let

$$f(x) = \begin{cases} x^2 + x & \text{if } x \geq -1; \\ k^2 + kx & \text{if } x < -1. \end{cases}$$

Find the value or values of k for which f is continuous at $x = -1$. You MUST use limits to justify your answers.

[5] 4. If f is differentiable, prove that

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x).$$

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[8] 5. Let y be a function of x which satisfies the equation $2x + y - \sqrt{2}\sin(xy) = \frac{\pi}{2}$. Find the slope of the tangent line to the function at the point $\left(\frac{\pi}{4}, 1\right)$.

[6] 6. Use only the definition of the derivative to find $f'(x)$ if $f(x) = \frac{1}{2x+1}$.

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[8] 7. A light sits on the ground 20m from a building. A man 2m tall walks away from the light directly toward the building at 1m/s. How fast is the length of his shadow on the building changing when he is 14m from the building?