Section 5.1: Areas & Distances

In this section we discover that in trying to find the area under a curve we end up with a special type of limit.

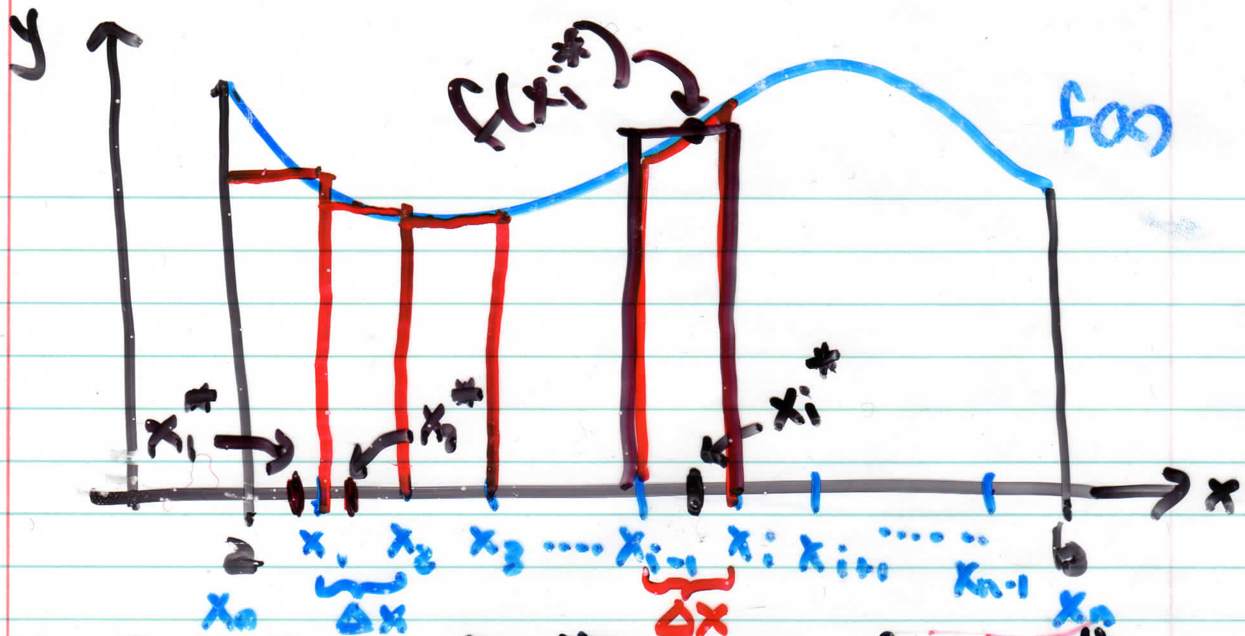
? What is the *Area Problem*?

We want to find the area of the region S that lies under a curve $y=f(x)$ $[f(x)\geq 0]$ from a to b.

It is easy to find the area of a region with straight sides. We just split it into rectangles & triangles & add up their areas.

Define Area: $A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} L_n$

We can make our defn' even more general by not specifying where in the interval we choose to draw the rectangle from (randomly) & by letting the area be bounded by a general fcn' $y=f(x)$ & the lines $x=a$ & $x=b$.



Now, we approximate the area of the i^{th} strip S_i with a rectangle with width Δx & height $\underline{f(x_i^*)}$, where $f(x_i^*)$ is the value of f at any number in the i^{th} subinterval $[x_{i-1}, x_i]$. We call these numbers $x_1^*, x_2^*, \ldots, x_n^*$ "sample points." So our generalized expression:

Section 5.2: The Definale Integral

Defn': If f is a cts. fcn' defined for $a \leq x \leq b$, we divide the interval $[a,b]$ into n subintervals of equal width $\Delta x = \frac{b-a}{n}$. Let $x_0 = a$ & $x_n = b$ be the endpoints of these subintervals & let $x_1^*, x_2^*, \ldots, x_n^*$ be any sample points in these subintervals, so x_i^* lies in the i^{th} subinterval $[x_{i-1}, x_i]$. [Then the "definite integral" of f from a to b is:

So we can interpret $\int_a^b f(x)dx$ as the area under the curve $f(x)$, above the x-axis, from $x=a$ to $x=b$.

Comparison Properties:

⑦ If $f(x) \geq 0$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx \geq 0$

⑧ If $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

⑨ If $m \leq f(x) \leq M$ for $a \leq x \leq b$, then

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

Section 5.3: Fundamental Theorem of Calculus

we start by looking at a fcn' defined by

$g(x) = \int_a^x f(t)\,dt$, where f is cts. on $[a,b]$ &

x varies between a & b. g depends only

on x. If x is fixed, $\int_a^x f(t)\,dt$ is a number.

If we let x vary, $\int_a^x f(t)\,dt$ also varies

& defines a fcn' of x denoted by $g(x)$.

Fundamental Thm', Part 1:

If f is cts on $[a,b]$, then the fcn' defined by $g(x)=\int_a^x f(t)\,dt$, $a \leq x \leq b$ is cts. on $[a,b]$ & differentiable on (a,b) & $g'(x)=f(x)$.

Fundamental Thm' Part 2:

If f is cts on $[a,b]$, then $\int_a^b f(x)dx = F(b) - F(a)$ where F is any antiderivative of f (ie, $F' = f$).

Now we know $\int$ represents antideriv!!

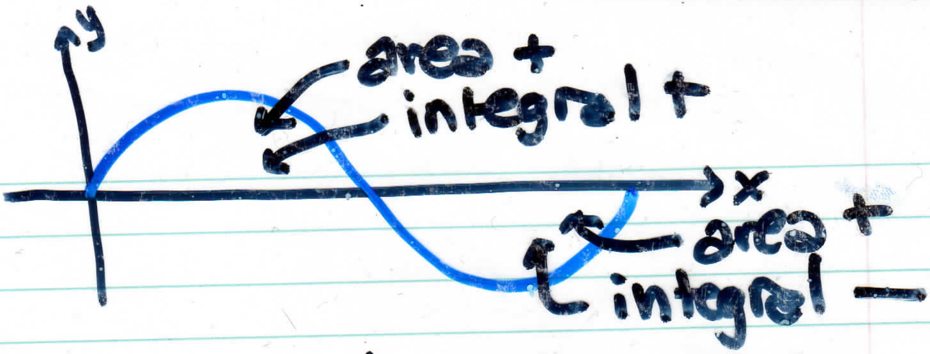
(4.10)

ex// Evaluate the integral $\int_1^3 e^x dx$.

ex// Evaluate the integrals

a) $\int_{-2}^{8} \sqrt[3]{x}\, dx$

We know



Now that we know how to solve integrals using antiderivatives, we can find different areas using integrals!

ex// Find the area by using integrals

a)