Chapter 4: APPLICATIONS OF DIFFERENTIATION

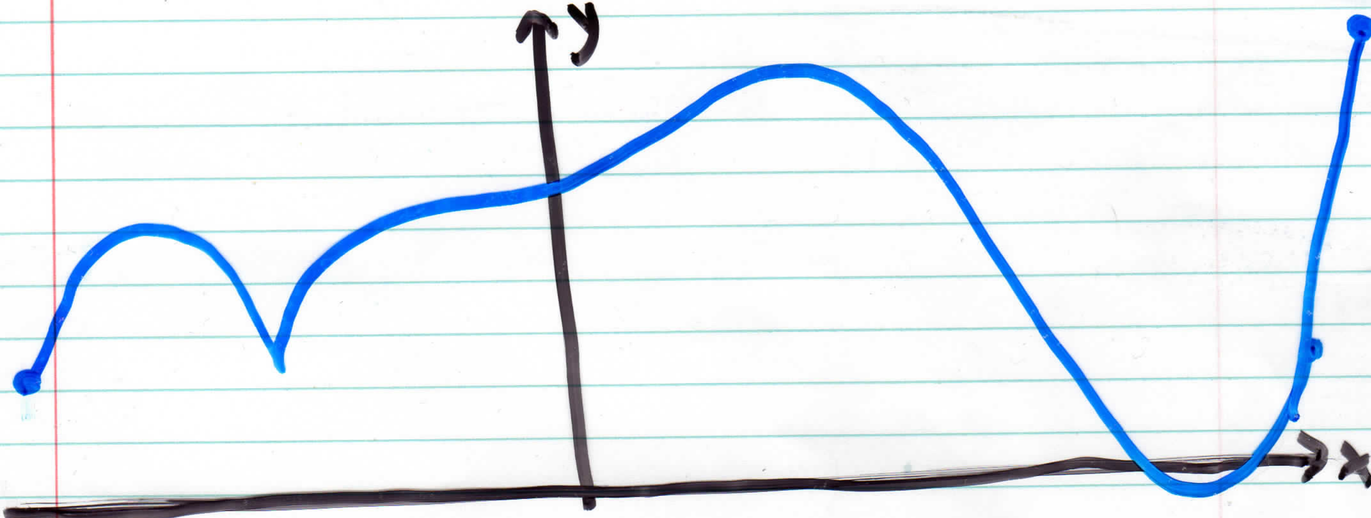
Section 4.1: Maximum & Minimum Values

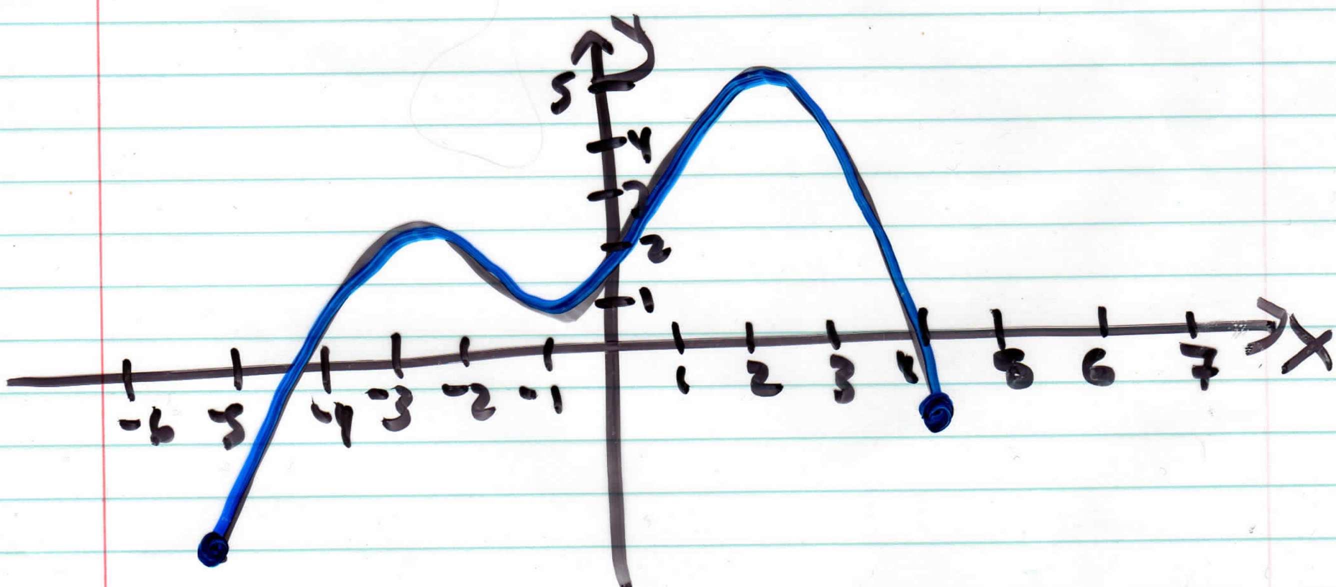
Some of the most important applications of the calculus of derivatives are called "optimization problems", where we need to find the optimal (best) way of doing something

Defn (1): A fcn' f has an "absolute (global) maximum" at c if $f(c) \geq f(x)$ for all x in D, where D is the domain of f. The number $f(c)$ is called a "maximum value" of f on D. Similarly, f has an "absolute (global) minimum" at c if $f(c) \leq f(x)$ for all x in D & $f(c)$ is called a "minimum value" of f on D. The maximum & minimum values of f are called the "extreme values" of f.

Defn'(2). A fcn' f has a "local maximum" (or "relative max") at c if $f(c) \geq f(x)$ when x is near c. Similarly, f has a "local (relative) min" at c if $f(c) \leq f(x)$ when x is near c.

both sides





We have seen that some functions **have** extreme values, & others (like x^3) do not.

Extreme Value Theorem: If f is $\boxed{\text{cts.}}$ on a $\boxed{\text{closed interval}}$ $[a,b]$, then f attains an abs. max value of $f(c)$ at some $x=c$ & an abs. min " " $f(d)$ " " $x=d$ (in $[a,b]$).

If we take a cts. fcn' that had no extreme values & restrict the interval we are looking at, it will! (by the Ex. V. Thm').

ex// what are the max/min values of $f(x)=x^3$ on $[-1, 1]$

The Extreme Value Thm' says that a cts. fcn' on a closed interval has an ABS max value & a ABS min value, but not how to find these extreme values.

Fermat's Thm': If f has a local max or min at c, & if $f'(c)$ exists, then $f'(c)=0$.

Section 4.2: The Mean Value Theorem

Now we know that at the number c where $f'(c)=0$, we may have a max or min. value. In this section we see how to tell whether a function will have any points that satisfy $f'(c)=0$.

Rolle's Theorem: Let f be a fcn' that satisfies the following 3 hypotheses:

1) f is cts. on the closed interval $[a,b]$

2) f is differentiable on (a,b)

3) $f(a)=f(b)$

Then there is a number c in (a,b) such that $f'(c)=0$.

Mean Value Thm': Let f be a fcn' that satisfies:

1) f is cts. on the closed interval $[a,b]$.

2) f is differentiable on (a,b).

Then there is a number c in (a,b) such that

$$\boxed{f'(c)=\frac{f(b)-f(a)}{b-a}}$$

or// $\underline{f(b)-f(a)=f'(c)(b-a)}$

This tells us that there is some point c where the slope of the tangent line $[f'(c)]$ = the slope of the secant line.

The M.V.T. lets us obtain info. about a fcn' from info about its derivative

ex// Suppose $f(0)=-3$ & $f'(x)\leq 5$ for all values of x. How large could $f(2)$ be?

Section 4.3: How Derivatives Affect the Shape of a Graph

The derivative can tell us where a fcn'
is increasing or decreasing.

Inc/Dec Test:

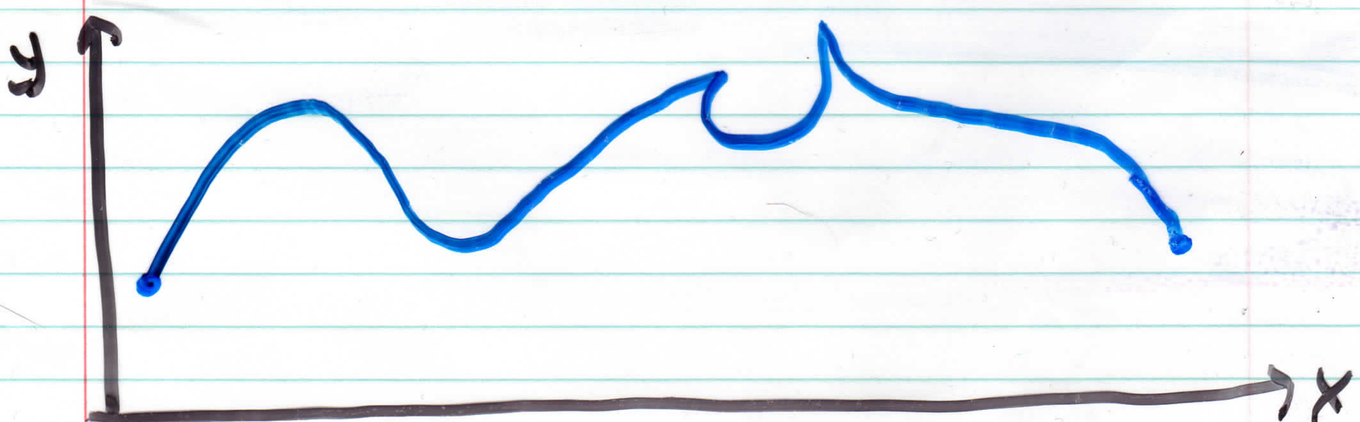
$[a,b]$

a) If $f'(x) > 0$ on an interval, then f is increasing.

b) If $f'(x) < 0$ " " " , " " " decreasing

Concavity Test:

a) if $f''(x) > 0$ for all x in I, f is CU on I.
b) " $f''(x) < 0$ " " ", f is CD on I.



Where f changes from CU to CD, there is what we call an "inflection point" (IP).

What are the possible shapes of graphs based on info from f' & f''?

f'' \ f'	+, +	+, −	−, +	−, −
+ (CU)				
− (CD)				

ex// Sketch a possible graph of a function f that would satisfy the following:

i) $f'(x)>0$ on $(-\infty,1)$ & $f'(x)<0$ on $(1,\infty)$

ii) $f''(x)>0$ on $(-\infty,-2)$ & $(2,\infty)$

$f''(x)<0$ on $(-2,2)$

iii) $\lim_{x\to-\infty} f(x)=-2$ & $\lim_{x\to\infty} f(x)=0$

Second Derivative Test:

a) If $f'(c)=0$ & $f''(c)>0$, then f has a local min at $x=c$.

b) If $f'(c)=0$ & $f''(c)<0$, then f has a local max at $x=c$.

c) If $f'(c)=0$ & $f''(c)=0$ or $f''(c)$ d.n.e, the test is inconclusive.

* The first deriative test tells us the same thing, so we have a choice as to which to use *

Section 4.5: Summary of Curve Sketching

You may ask, why all of these methods when I could just plot points & connect?

GUIDLINES FOR CURVE SKETCHING

① Domain: check where $f(x)$ is undefined

② Intercepts: y Intercepts → set $x=0$
x intercepts → set $y=0$

③ Symmetry (optional): check if $f(-x) = \underbrace{f(x)}_{\text{even}}$ or $\underbrace{-f(x)}_{\text{odd}}$

④ Asymptotes: H.A. → check $\lim_{x\to\pm\infty} f(x)$ for $y = $ H.A.
V.A. → check $\lim_{x\to a} f(x) = \infty$ for $x = $ V.A.
← for places where $f(a)$ is undefined.

⑤ Inc/Dec: Compute f' & find the intervals where $\overset{(INC)}{f'(x)>0}$ & $\overset{(DEC)}{f'(x)<0}$, & critical numbers where $f'(x)=0$ or $f'(x)$ d.n.e.

⑥ Local Max/Mins: Check your critical numbers from ⑤ to see if they are max/mins.

⑦ Concavity: Compute f'' & find the possible inflection points. Check where $\overset{(CU)}{f''(x)>0}$ & $\overset{(CD)}{f''(x)<0}$ & confirm inflection points.

⑧ Sketch !! (label important points first).