Section 4.7: Optimization Problems

The methods we have learned in Ch.4 for finding extreme values have practical applications in many areas of life.
For example, minimize cost, time, or distance, or maximize profit, area, volume, etc.

The problems we encounter are word problems similar to related rates problems, so our strategy for solving them is similar:

1. Read carefully, what do I know? what do I need?
2. Draw a diagram
3. Assign values to the variables involved
4. Decide which variable is to be maximized/minimized & express it in terms of the others.
5. Use what we know to eliminate all but 1 variable in the eqn' 4. *
6. Write the domain of *

7 Use the methods from 4.1 & 4.3 to find the ABSOLUTE max/min of *

ex// A farmer has <u>2400 ft</u> of fencing & wants to fence off a rectangular field that borders a straight river (so he needs no fence along the river). What are the dimensions of the field that has the largest area?

Q4 A cylindrical closed can is to be made to hold 1L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

e.g. Find the area of the largest rectangle that can be inscribed in a semi-circle with radius r.

Section 4.10: Antiderivatives

Say we know the velocity of a particle, but want to know its position at a given time, or, we know the rate at which bacteria is growing, but want to know the size at a given time. In other words, GIVEN THE DERIVATIVE, can we work backwards ??

A function F is called an "antiderivative" of $f(x)$ on an interval I if $f(x) = F'(x)$ in I (ie, $f(x)$ is the derivative, F is the original)

So if F & G are any 2 antiderivatives of f, then $F'(x) = f(x) = G'(x) \Rightarrow G(x) - F(x) = C$ (they must differ only by their constants).

<u>Thm'</u>: If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is $F(x) + C$

By assigning specific values to C, we obtain a family of functions whose graphs are vertical translates of one another.

ex// Members of the family of anti-derivatives of $f(x) = x^2$

Table of Antiderivatives: (Particular, ie, C=0)

Fcn'	Anti-Deriv	Fcn'	Anti-Deriv.
$(n\neq -1)$ x^n	$\frac{x^{n+1}}{n+1}$	$\cos x$	$\sin x$
$\frac{1}{x}$	$\ln\lvert x\rvert$	$\sin x$	$-\cos x$
e^x	e^x	$\sec^2 x$	$\tan x$
		$\sec x \tan x$	$\sec x$

Antiderivative Rules:

1. $cf(x)$ has antiderivative $cF(x)$.

2. $f(x) \pm g(x)$ has antiderivative $F(x) \pm G(x)$.

ex/ Find <u>all</u> functions $g(x)$ such that

$$g'(x) = 4\sin x + \frac{2x^5 - \sqrt{x}}{x}$$

In applications of calculus, it is very common to have a situation where it is required to find a function, given knowledge about its derivative. An eqn' that involves the derivatives of a fcn' is called a "differential equation".

? Can we ever determine C specifically ?

In some cases there may be some extra conditions that determine the constants & therefore uniquely specify the soln!

Notice: When we started with f'', we needed 2 pieces of info. because we ended with 2 unknown constants. In fact:

Rectilinear Motion:

Recall that if an object has position fcn' $s=f(t)$, velocity is $v(t)=s'(t)$. So position is the antiderivative of velocity. Likewise, acceleration $a(t)=v'(t)=s''(t)$ so velocity is the antiderivative of acceleration. So, now we can find $v(t)$ or $s(t)$ from $a(t)$!