Section 1.6: Inverse Functions & Logarithms

Lets say we observe a population of bacteria. The size of the bacteria is recorded hourly. So, the number of bacteria N is a fcn' of time, $N=f(t)$. Say instead we want to study the time required for the population to reach various levels. We are now thinking of t as a fcn of N. This fcn' is the "inverse" function f^{-1}. So $t=f^{-1}(N)$.

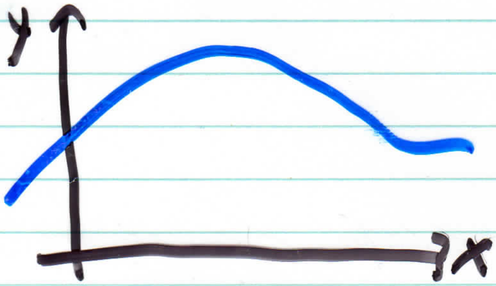
So the <u>domain</u> of f is the <u>range</u> of f^{-1}

& " <u>range</u> of " " " <u>domain</u> " "

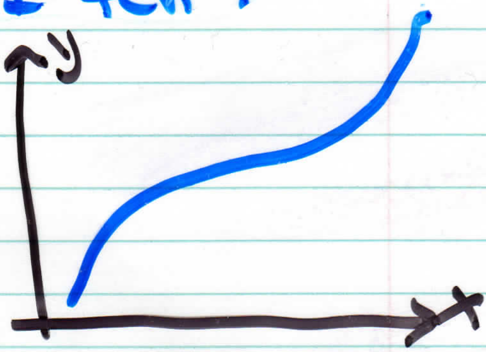
<u>recall</u>: We used the "vertical line test" to distinguish fcn's from non-fcn's. To test if a fcn' is 1:1, we can use the "horizontal line test".

ex// Is the following a 1:1 fcn'?

a)



b)



ex// Is the function 1:1?

a) $f(x)=x^3$

b) $f(x)=x^2$

Logarithmic Fcn's: If $a>0$ & $a\neq 1$, then the function $f(x)=a^x$ is 1:1

$\Rightarrow$ it has an inverse (f^{-1})

called the "logarithmic function with base a"

$$\boxed{\log_a x = y \Leftrightarrow a^y = x}$$

Logarithm Laws:

If x & y are positive numbers, then:

① $\log_a(xy) = \log_a x + \log_a y$

② $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

③ $\log_a(x^r) = r\log_a x$

④ $\log_a x = \frac{\ln x}{\ln a}$

ex// Evaluate $\log_2 80 - \log_2 5$

Of all possible bases for log, the base e is the most convenient. We call it the "natural logarithm". $\log_e x = \ln x$

We can use logarithms to "bring down" variables in an exponent, so we can solve.

ex// Solve for x.

a) $\ln x = 5$

b) $e^{5-3x} = 10$

c) $2\ln x = 1$

To relate $\log_a$ to $\ln$, we have $\log_a x = \frac{\ln x}{\ln a}$

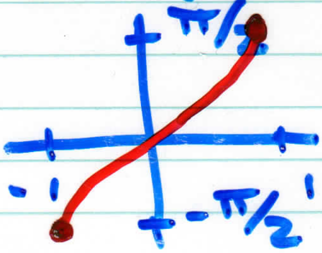
Finally, we have inverse trig. fcn's.

But notice sin, cos, & tan are NOT 1:1!

so we must restrict their domains.

① $\sin^{-1}x = y \Leftrightarrow \sin y = x$ on $-\frac{\pi}{2} \leq y \leq \pi/2$

"arcsin"

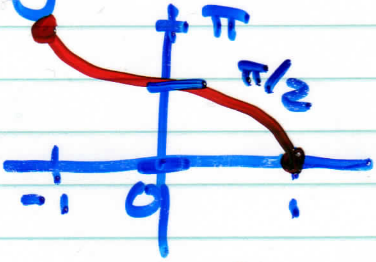


domain $[-1, 1]$

range $[-\pi/2, \pi/2]$

② $\cos^{-1}x = y \Leftrightarrow \cos y = x$ on $0 \leq y \leq \pi$

"arccos"

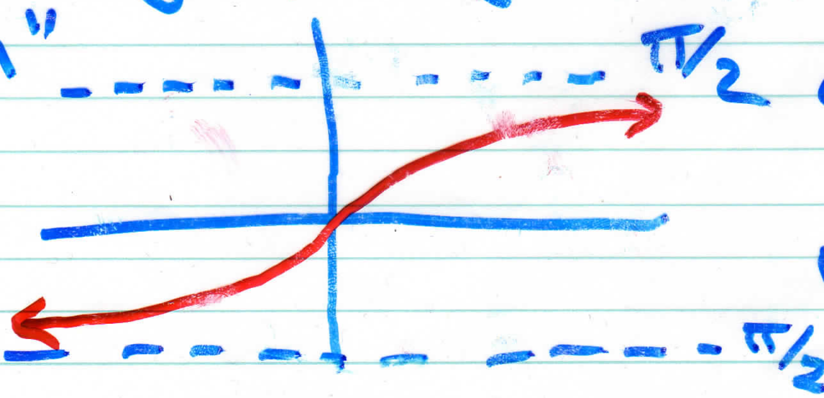


domain $[-1, 1]$

range $[0, \pi]$

③ $\tan^{-1}x = y \Leftrightarrow \tan y = x$ on $-\pi/2 < y < \pi/2$

"arctan"



domain $(-\infty, \infty)$

range $(-\pi/2, \pi/2)$

Section 3.8: Derivatives of Log. Functions

Now that we are familiar with Log. fcn's, we can find their derivatives.

① $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

② $\frac{d}{dx}(\ln x) = \frac{1}{x}$

③ $\frac{d}{dx}(\log_a (g(x))) = \frac{g'(x)}{g(x) \ln a}$

④ $\frac{d}{dx}(\ln g(x)) = \frac{g'(x)}{g(x)}$

⑤ $\frac{d}{dx}(a^x) = a^x \cdot \ln a$

⑥ $\frac{d}{dx}(a^{g(x)}) = a^{g(x)} \cdot \ln a \cdot g'(x)$

Sometimes, the calculation of derivatives of complicated functions can be made easier by taking logarithms. Steps:

1. take natural log's (ln) of both sides of an eqn' of the form $y=f(x)$ & use the Logarithm Laws to simplify it.
2. Use implicit differentiation with respect to x.
3. Solve for y' (this is the derivative we were looking for!)