Chapter 3: DIFFERENTIATION RULES

Section 3.1: Derivatives of Polynomials & Exponentials

We can find derivatives in a faster way than using the limit defn' of a derivative with these formulas:

① $\frac{d}{dx}(c) = 0$
constant

② $\frac{d}{dx}(x^n) = nx^{n-1}$

ex// find the derivatives of the following

a) $f(x) = x^6$

b) $f(x) = x^{1000}$

c) $f(x) = \frac{1}{x}$

d) $f(x) = \sqrt{x}$

e) $f(x) = \sqrt[3]{x^4}$

③ $\frac{d}{dx}(e^x) = e^x$

ex// what is the slope of the tangent to $y = e^x$ at any point?
$x = a$

Section 3.2: The Product & Quotient Rules

Suppose we have 2 functions, $f(x) = x$ & $g(x) = x^2$.

We can form their product $f(x) \cdot g(x) = fg = (x)(x^2) = x^3$

& their quotient $\frac{f}{g} = \frac{x}{x^2} = \frac{1}{x} = x^{-1}$

Section 3.4: Derivatives of Trig. Functions

Let's start with a review of trigonometry:

ex// Differentiate:

Trig. functions are often used in modeling real-world phenomena. In particular; vibrations, waves, elastic motions & other quantities that vary in a periodic manner.

ex// An object at the end of a vertical spring is stretched 4 cm beyond its rest position & released at time $t=0$. Its position at time t is

$$s = f(t) = 4\cos t$$

Find the velocity at time t & use it to analyze the object's **motion**.

Section 3.5: Chain Rule

So far, we can calculate the derivatives of most functions. However, we have not yet seen how to find the derivative of a function that is *inside* another function.

ex/

So, if f & g are both differentiable & $F = f \circ g$ is the composite function defined by $F(x) = f(g(x))$, then F is differentiable & F′ is given by:

$$\boxed{F'(x) = \underbrace{f'(g(x))}\cdot\underbrace{g'(x)}}$$

All we need to do is figure out which is the *inside* function & which is the *outside* function.

STEPS FOR DERIVATIVES

① Which rule(s) do I need to use?

a) $\frac{d}{dx}(c) = 0$

b) $\frac{d}{dx}(x^n) = nx^{n-1}$ POWER RULE

c) $\frac{d}{dx}(e^x) = e^x$

d) PRODUCT RULE: $(fg)' = f'g + g'f$

e) QUOTIENT RULE: $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$

f) TRIG. RULES: $\frac{d}{dx}(\sin x) = \cos x$, $\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$, $\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$, $\frac{d}{dx}(\cot x) = -\csc^2 x$

g) CHAIN RULE: $[f(g(x))]' = f'(g(x)) \cdot g'(x)$

$\rightarrow [e^{(\text{anything})}]'$ or $[e^{f(x)}]' = e^{f(x)} \cdot f'(x)$

(identify which is the *inside*, which is *outside*).

When in doubt, write down the rule you need

Section 3.6: Implicit Differentiation

So far, all of the functions we have been working with have been of the type $y=f(x)$, where we have one variable in terms of another

Some functions, however, are defined by a *relation* between x & y

The first type of functions are called "explicit" & the second are called "implicit". In some cases we can re-arrange an implicit function to get an explicit one:

In some cases, we can not:

For these types of fcn's or fcn's where solving for y brings complications, we need a new way to find their derivatives y' (or $\frac{dy}{dx}$). This consists of differentiating both sides of an eqn' with respect to x & solving for y'.

Remember, in our cases so far, y was the dependent variable ($y=f(x)$ so y depended on x) & x was the independent variable.

Our variables do not always need to be called y & x. As long as we know which depends on which, we can use implicit differentiation to find derivatives.

ex/ A population of bees (p) grows as time (t) goes on. The growth of its population can be described by the equation: $e^{pt} = t + p$

Find the rate of change of the population.

or// we can have more than 1 dependent variable.

ex// Suppose a circle expands as time goes on. The area of a circle is given by:

Section 3.7: Higher Derivatives

If f is a differentiable function, then its derivative is also a function, so f' may have a derivative of it's own!

In general, we can interpret the second derivative as a rate of change OF a rate of change. If the 1^{st} deriv. is velocity, then the 2^{nd} deriv. is "acceleration," $a(t)$.

$$a(t) = v'(t) = s''(t)$$

ex// The position of a particle is given by the equation $s = f(t) = t^3 - 6t^2 + 9t$ (t - seconds, s - meters).
Find the acceleration at time t. What is the acceleration after 4 seconds?

ex. Find y'' if $x^4+y^4=16$

Section 3.10: Related Rates

In this section we see (word) problems where we need to compute the rate of change of one quantity in terms of the rate of change of another quantity.

The process is to find an eqn' that relates the two or more quantities & then to differentiate both sides with respect to time.

ex// Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50cm?

ex/ A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft. away from the wall?

ex// A water tank has the shape of an inverted circular cone with base radius 2 m & height 4 m. If water is being pumped into the tank at a rate of $2\text{m}^3/\text{min}$, find the rate at which the water level is rising when the water is 3 m deep?

ex// Car A is traveling west at 50 mph & car B is traveling north at 60 mph. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 miles & car B is 0.4 miles from the intersection?

ex 11 A man walks along a straight path at a speed of 4 ft/s. A searchlight is located on the ground 20 ft from the path & is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?

ex// A plane flying horizontally at an altitude of 1 mile & a speed of 500 mph passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 miles away from the station.