CHAPTER 1: FUNCTIONS & MODELS

Section 1.1: Four Ways to Represent a Function

The fundamental objects that we deal with in calculus are called "**functions**" (fcn's).
They arise whenever one quantity depends on another. Consider the following:

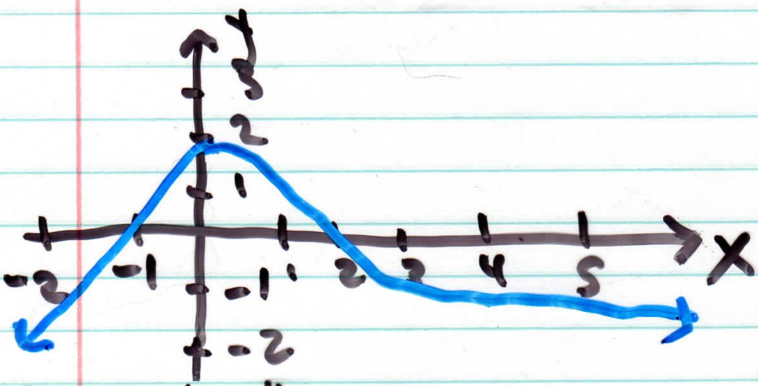
There are four possible ways to represent a function:

ex// When you turn on a hot-water faucet, the temperature T of the water depends on how long the water has been running. Draw a rough graph of T as a function of the time t that has elapsed since the faucet was turned on.

ex// A rectangular storage container with an open top has a volume of $10m^3$. The length of its base is twice its width. Material for the base costs $\$10/m^2$ & $\$6/m^2$ for the sides. Express the cost of materials as a function of the width of the base.

<u>Defn'</u>: A "*function*" f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B.

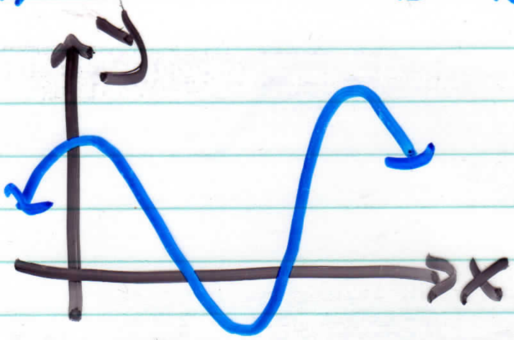
ex// Below is the graph of a function f(x). Find the values of f(-1), f(0), & f(3).



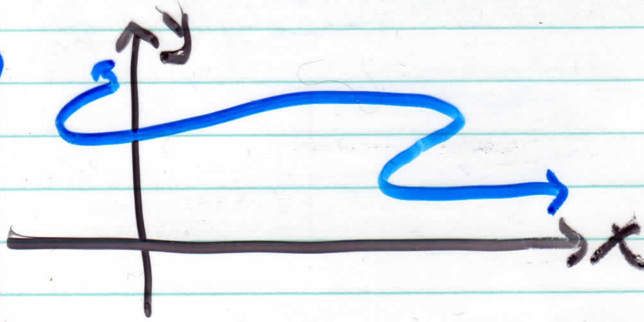
note: Not all curves in the x-y plane are the graphs of functions! A curve in the x-y plane is the graph of a function of x if and only if no vertical line intersects the curve more than once (the "vertical line test").

ex// Which of the following are functions?

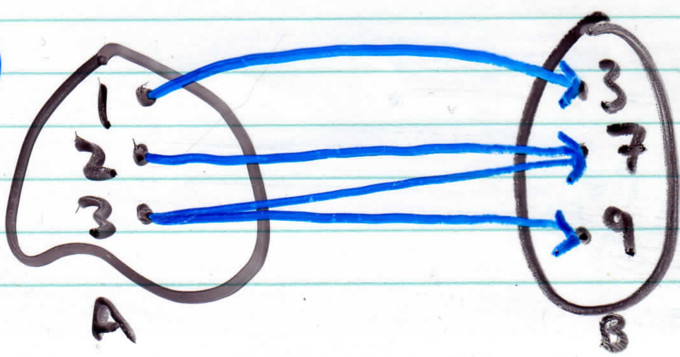
a)



b)



c)



d)

e) $y^2 = x$

Features of a Function:

We usually consider functions for which the sets A & B are sets of real numbers.

The **DOMAIN** of f is the set of all possible values where f(x) is defined.
The **RANGE** of f is of all possible values of f(x) as x varies through the domain.
The values in A are the "independent variables"
" " " B " " "dependent variables"

Type of Fcn'	Domain
Polynomial Fcn' constants $p(x) = a_0 x + a_1 x^2 + \cdots + a_n x^{n+1} + \cdots$ ex:	$x \in \mathbb{R}$ $(-\infty, \infty)$
Rational Fcn' $\frac{p(x)}{q(x)}$ polynomials ex:	all x except those which make $q(x) = 0$
Root Fcn' $\sqrt{p(x)}$ ex:	$p(x) \geq 0$

Piecewise Defined Fcn's: Some functions are defined by different formulas in different parts of their domains.

ex// Find the domain & sketch a graph of

$$f(x) = \begin{cases} 2x+3 & \text{if } x < -1 \\ 3-x & \text{if } x \geq -1 \end{cases}$$

ex// The "absolute value function" $|x|$ (makes #'s positive)

Symmetry: There are 2 types of symmetries that we discuss when we talk about functions:

1) EVEN functions: these satisfy $\boxed{f(-x) = f(x)}$ & are symmetric about the y-axis.

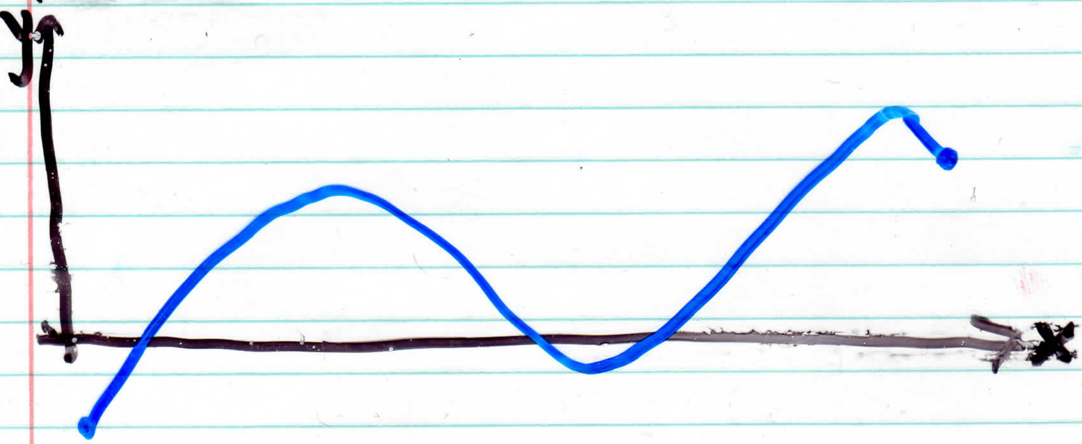
ex// $f(x) = x^2$

2) ODD functions: these satisfy $\boxed{f(-x) = -f(x)}$ & are symmetric about the origin (180°).

ex// $f(x) = x^3$

ex// Are the following fcn's odd, even, or neither?

Increasing/Decreasing: A function f is "increasing" on an interval I if $f(x_1) < f(x_2)$ for $x_1 < x_2$ in I. f is "decreasing" on I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I.



Section 1.3: New Functions from Old Functions.

Once we have a basic function, we can quickly sketch graphs & write equations for related functions by following some simple rules. Say we have a fcn' $y = x^2$

old fcn: $y=f(x)$

Rules: ① $y=f(x)+c \Rightarrow$ shift c units up $(c>0)$

② $y=f(x)-c \Rightarrow$ shift c units down $(c>0)$

③ $y=f(x-c) \Rightarrow$ shift c units right $(c>0)$

④ $y=f(x+c) \Rightarrow$ shift c units left $(c>0)$

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⑤ $y=cf(x) \Rightarrow$ stretch vertically by c $(c>1)$

⑥ $y=\frac{1}{c}f(x) \Rightarrow$ compress vertically by c $(c>1)$

⑦ $y=f(cx) \Rightarrow$ compress horizontally by c $(c>1)$

⑧ $y=f(\frac{1}{c}x)=f(\frac{x}{c}) \Rightarrow$ stretch horizontally by c $(c>1)$

⑨ $y=-f(x) \Rightarrow$ reflect about the x-axis

⑩ $y=f(-x) \Rightarrow$ reflect about the y-axis

Combinations of Functions: we can combine two functions $f(x)$ & $g(x)$ to form 4 new functions:

If the domain of f was A & the domain of g was B, then the domain of any of these new functions is just the intersection of A & B (& where $g(x) \neq 0$ for 4)

ex//

ex// Find the functions $f+g$, $f-g$, fg, & f/g & their domains if $f(x)=\sqrt{x}$ & $g(x)=1-x$

Composition of Functions: There is another way of combining two functions to get a new function.

It is called "composition", & is like putting one function *inside* the other. We write it as $(f \circ g)(x) = f(g(x))$ & its domain is where both $g(x)$ & $f(g(x))$ are defined.

ex// Find the composite functions $f \circ g$ & $g \circ f$ & their domains if $f(x) = x^2$ & $g(x) = x - 3$

We can also compose a function with itself

ex// find $g \circ f$, $f \circ g$, $f \circ f$, $g \circ g$, & their domains if $f(x) = \sqrt{x}$ & $g(x) = \sqrt{2-x}$

Section 1.5: The Exponential Function

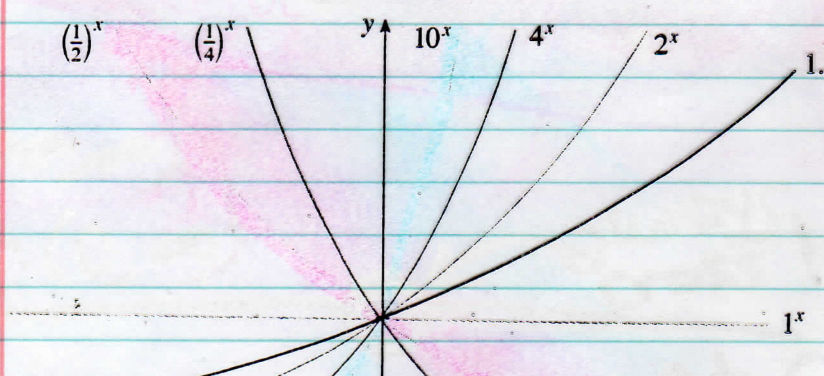
The "exponential function" is a fcn' of the form.

$$f(x) = a^x$$

(It should not be confused with the power function x^n (ie, x^2) where the variable is the base)

How can we figure out what a^x is if x is an irrational number (doesn't make a fraction, like $\sqrt{2}$, or π)

ex// what is $2^{\sqrt{3}}$?



The exponential fcn' occurs frequently in mathematical models of nature & society, In particular, in the descriptions of population growth & radioactive decay.

ex// The half-life of stronium-90 (^{90}Sr) is 25 years. This means half of any given quantity of ^{90}Sr will disintigrate in 25 years.

a) If a sample of ^{90}Sr has a mass of 24 mg, find an expression for the mass $m(t)$ that remains after t years.

b) Find the mass remaining after 40 years e: of all possible bases for an exponential function, there is one that is most convenient for calculus purposes. We call this number "e".

Chapter 2: LIMITS & DERIVATIVES

Section 2.2: The Limit of a Function

What is the value of $f(x) = \frac{x-1}{x^2-1}$ at $x=1$?