**UNIVERSITY OF MANITOBA**

DATE: December 19, 2005 — FINAL EXAMINATION

PAPER # 514 — TITLE PAGE

DEPARTMENT & COURSE NO: 136.130 — TIME: 2 hour

EXAMINATION: Vector Geometry and Linear Algebra — EXAMINERS: Various

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FAMILY NAME: (Print in ink) ______________________

FIRST NAME: (Print in ink) ______________________

STUDENT NUMBER: ______________________

SEAT NUMBER: ______________________

SIGNATURE: (Print in ink) ______________________

(I understand that cheating is a serious offense)

**Please indicate your instructor and section by checking the appropriate box below:**

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|---|---|---|---|---|
| ☐ | L01 | G.I. Moghaddam | M,W,F | 9:30 - 10:20 |
| ☐ | L02 | J. Arino | Tues, Thurs | 8:30 - 9:50 |
| ☐ | L03 | G.I. Moghaddam | M,W,F | 1:30 - 2:20 |
| ☐ | L04 | N. Zorboska | Tues, Thurs | 11:30 - 12:50 |
| ☐ | L91 | Challenge for Credit | | |
| ☐ | SJR | | | |

**INSTRUCTIONS TO STUDENTS:**

This is a 2 hour exam. **Please show your work clearly**.

**No texts, notes, or other aids are permitted. No calculators, cellphones or electronic translators permitted.**

This exam has a title pages, 9 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staples.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 100 points.

**Answer all questions on the exam paper** in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

| Question | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 6 | |
| 3 | 12 | |
| 4 | 7 | |
| 5 | 8 | |
| 6 | 15 | |
| 7 | 11 | |
| 8 | 11 | |
| 9 | 8 | |
| 10 | 12 | |
| Total: | 100 | |

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[10] 1. Each of the following matrices is the augmented matrix of a linear system. Complete the table for each system.

$$A = \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right], \quad B = \left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right], \quad C = \left[\begin{array}{cccc|c} 1 & -1 & 2 & 0 & 4 \\ 1 & -1 & 2 & 0 & 5 \\ 0 & 0 & -2 & 0 & 4 \end{array}\right]$$

| Augmented matrix | number of equations | number of variables | number of solutions | number of parameters (if applicable) |
|---|---|---|---|---|
| A | | | | |
| B | | | | |
| C | | | | |

[6] 2. If $det \begin{bmatrix} 1 & -2 & 4 \\ a & b & c \\ 3 & 5 & -6 \end{bmatrix} = -4$, use properties of determinant to evaluate

$$det \begin{bmatrix} 1 & -2 & 4 \\ 3 & 5 & -6 \\ 2(a-1) & 2(b+2) & 2(c-4) \end{bmatrix}.$$

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[12] 3. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and let $B$ be a $3 \times 3$ matrix with $det(B) = -2$. Find each of the following:

(a) $det\,(2A^8B^{-1})$

(b) $det\left(ADBD^{-1}\right)$ (where $D$ is a $3 \times 3$ matrix)

(c) The numbers a, b, and c, such that

$$adj(A) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & a \\ 0 & b & c \end{bmatrix}$$

(d) $A^{-1}$

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[7] 4. Let $A = \begin{bmatrix} 1 & 1 & 6 & 0 \\ 0 & 1 & 2 & 1 \\ -1 & 0 & -1 & 2 \\ 0 & 3 & 0 & 0 \end{bmatrix}$. Use Cramer's Rule to solve the linear system

$$A \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 2 \\ 0 \end{bmatrix}$$ **for z only**.

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[8] 5. Let $A = \begin{bmatrix} 1 & 0 & 1 & -4 \\ -1 & 3 & 5 & 6 \\ 2 & 4 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 & -4 \\ 2 & 4 & -1 & 1 \\ 0 & 3 & 6 & 2 \end{bmatrix}$

Find elementary matrices $E_1$ and $E_2$ such that $B = E_2 E_1 A$

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[15] 6. Let $\mathbf{u} = (2, -1, 0)$, $\mathbf{v} = (-1, 1, 1)$ and $\mathbf{w} = (1, 0, -1)$.
Find, by showing all your work:

(a) $proj_{\mathbf{v}}\mathbf{u}$.

(b) The area of the parallelogram determined by $\mathbf{u}$ and $\mathbf{v}$.

(c) The volume of the parallelepiped determined by $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$.

(d) All of the unit vectors that are parallel to $\mathbf{u}$.

(e) All vectors $\mathbf{x} = (\mathrm{a}, \mathrm{b}, \mathrm{c})$ that are orthogonal to $\mathbf{u}$.

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[11] 7. Two lines are given by their parametric equations:

$$l_1 : \begin{cases} x = 4 + t \\ y = -1 \\ z = 2 + t \end{cases} \quad \text{and} \quad l_2 : \begin{cases} x = 2 \\ y = 3 - 2s \\ z = -6 + 3s \end{cases} \quad \text{, for } t \text{ and } s \text{ in } \mathbb{R}.$$

(a) Find the point of intersection of $l_1$ and $l_2$.

(b) Find a vector orthogonal to both $l_1$ and $l_2$.

(c) Find an equation of the plane containing $l_1$ and $l_2$.

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[11] 8. Let $M_{22}$ denote the vector space of all $2 \times 2$ matrices and let $O$ denote the zero $2 \times 2$ matrix.

(a) If $B$ is a fixed $2 \times 2$ matrix, show that the set $W$ of all $2 \times 2$ matrices $A$ such that $AB = O$, i.e. $W = \{A \text{ in } \mathrm{M}_{22} : AB = O\}$, is a subspace of $M_{22}$.

(b) If $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, find a basis for the vector space $W$ in part (a).

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[8] 9. Let $\mathbf{u} = (1, 0, 1, 0)$, $\mathbf{v} = (1, -1, 1, 0)$, $\mathbf{w} = (1, 0, 0, 0)$.

(a) Is the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ linearly independent? Show your work.

(b) Does the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ span $\mathbb{R}^4$? Explain why.

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[12] 10. (a) Let $A$ be a $3 \times 7$ matrix. Answer the following questions by filling in the blanks:

i. The column space of $A$ is a subspace of $\mathbb{R}^n$ with $n$ equal to ____________.

ii. If the rows of $A$ are linearly independent, the dimension of the row space of $A$ is equal to ____________.

iii. If the rows of $A$ are linearly independent, the dimension of the column space of $A$ is equal to ____________ .

iv. If the rows of $A$ are linearly independent, the dimension of the null space of $A$ is equal to ____________ .

v. If $A$ is the zero $3 \times 7$ matrix, the dimension of the null space of $A$ is equal to ____________.

(b) If $B$ is a matrix such that its reduced row echelon form equals

$$\begin{bmatrix} 1 & -1 & 0 & -2 & 0 & 3 \\ 0 & 0 & 1 & 2 & 5 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

find a basis for the row space of $B$.

(c) If $C$ is a matrix such that its reduced row echelon form equals

$$\begin{bmatrix} 1 & 2 & 3 & 0 & -2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

find a basis for the null space of $C$. Show your work.