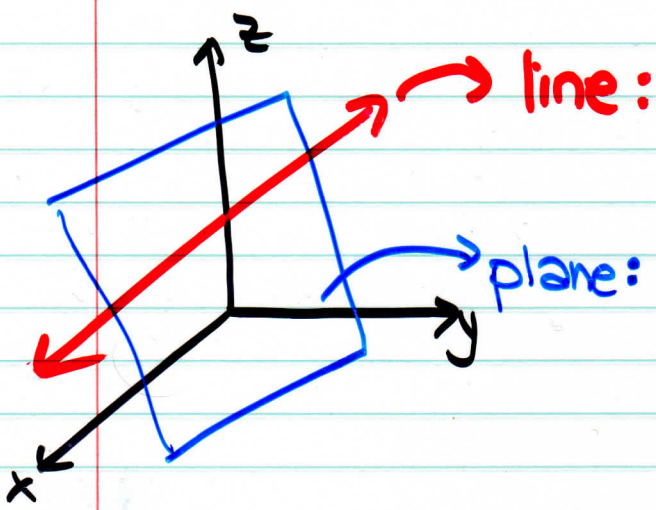
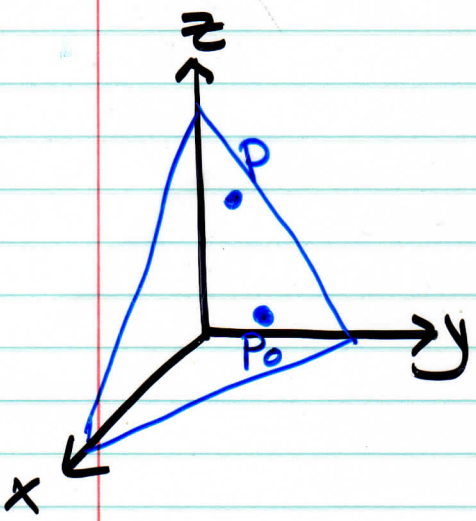
Section 3.5: Lines & Planes





ex: Find an eqn of the plane passing through the point $(1, -2, 3)$ & $\perp$ $\vec{n} = (7, -4, 1)$.

Thm: The graph of the eqn $ax+by+cz+d=0$ (where a, b, c, d not all 0) is the plane with normal vector $\vec{n} = (a, b, c)$.

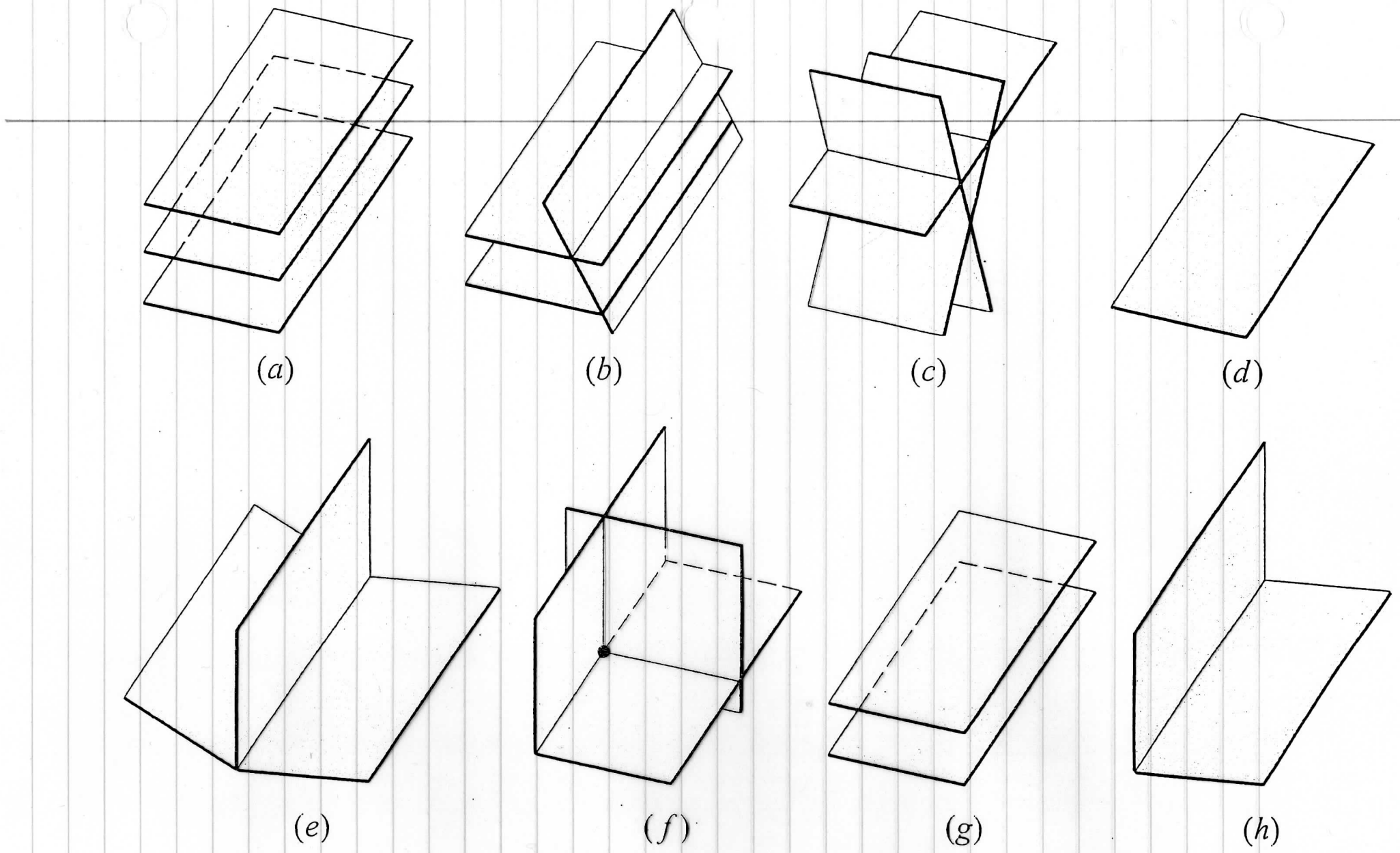


Figure 3.5.2 (*a*) No solutions (3 parallel planes). (*b*) No solutions (2 parallel planes). (*c*) No solutions (3 planes with no common intersection). (*d*) Infinitely many solutions (3 coincident planes). (*e*) Infinitely many solutions (3 planes intersecting in a line). (*f*) One solution (3 planes intersecting at a point). (*g*) No solutions (2 coincident planes parallel to a third plane). (*h*) Infinitely many solutions (2 coincident planes intersecting a third plane).

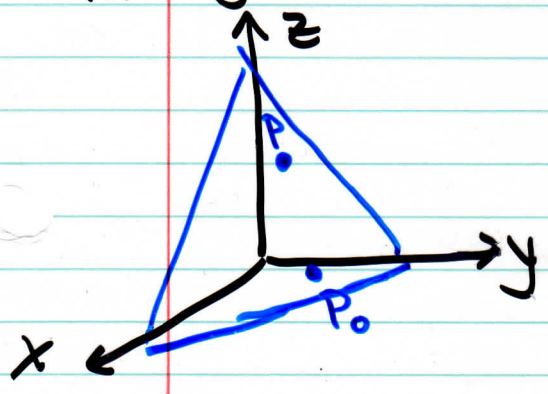
Just as the solution of a system of linear eqn's

$ax+by=k_1$
$cx+dy=k_2$ has soln's that correspond to the intersection of the lines...

$ax+by+cz=k_1$
$dx+ey+fz=k_2$
$gx+hy+iz=k_3$ corresponds to the intersection of...

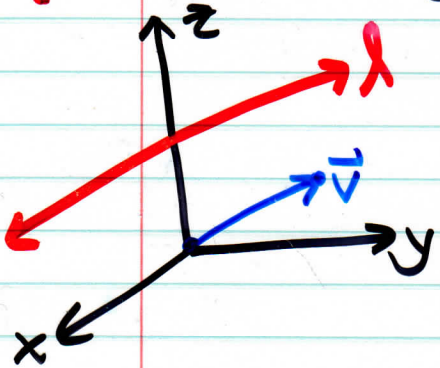
ex// Find the eqn of the plane through the points $P_1(1,2,-1)$, $P_2(2,3,1)$, & $P_3(3,-1,2)$

We can write the P.N. form of a plane nicely in terms of vectors!



ex// Find the vector eqn' of the plane that passes through the point $(6, 3, -4)$ & is $\perp \vec{n} = (-1, 2, 5)$.

? How do we get the eqn' for a line in 3-space ?



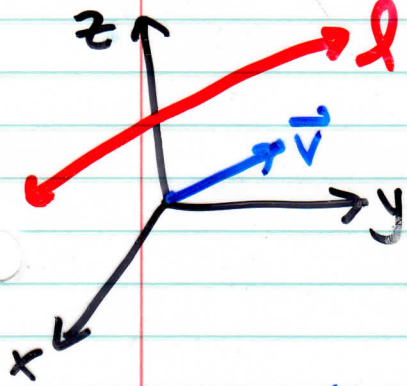
ex// a) Find parametric eqn's for the line l passing through $P_1(2, 4, -1)$ & $P_2(5, 0, 7)$.

b) where does the line intersect the x-y plane?

ex// Find parametric eqn's for the line of intersection of the planes

$3x+2y-4z-6=0$ & $x-3y-2z-4=0$.

We can also write a line in terms of vector eqns!

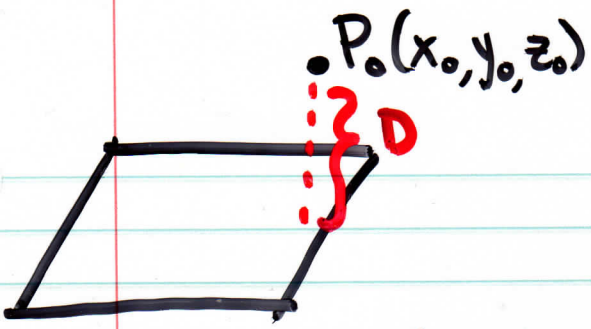


ex// What is the vector form eqn' for the line through $(1,-2,7)$ & parallel to the vector $\vec{v}=\langle 4,0,-3\rangle$?

There are 2 major "distance problems" in 3-space:

a) Find the distance between a point & a plane.

b) Find the distance between two parallel planes.



⇒ The distance b/w a point $P_0(x_0, y_0, z_0)$ & the plane $ax+by+cz+d=0$:

ex// What is the distance between the point $(4, 2, -1)$ & the plane $3x-y+5z=-4$?

ex// a) Verify that the planes $x+2y-2z=3$ & $2x+4y-4z=7$ are parallel.

b) What is the distance between the planes?