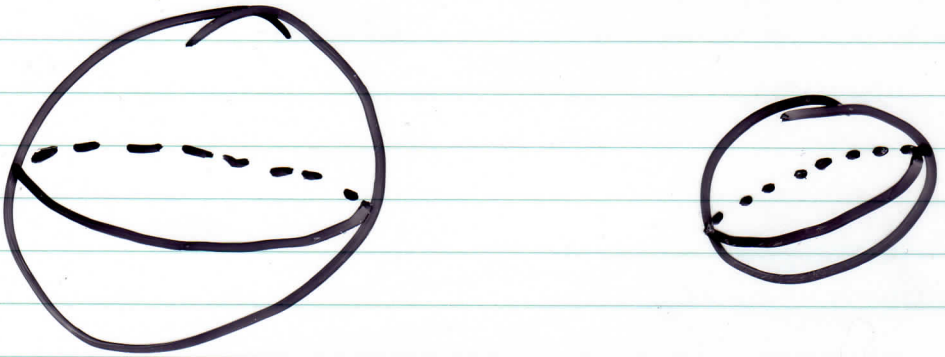
Chapter 6: Topology

Section 6.1: Topological Spaces

Topology is the study of mutual relationships of the parts of objects. While in geometry it is important how far a point is from another point, in topology it is not —

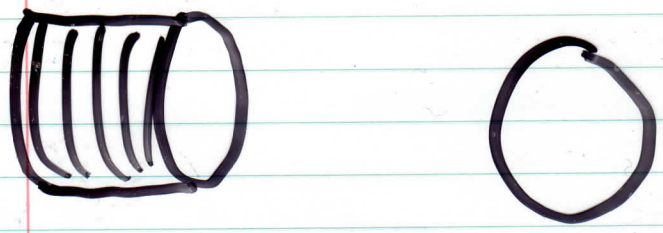


Any two objects having the same "non-geometric" (independent from distance & related properties are "homeomorphic".

It is customary in topology to discuss spaces other than objects.

However, homeomorphism is still unclear, & to understand a more precise definition we would need a mathematical background.

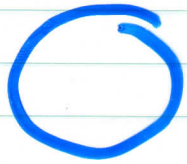
Consider any cylinder open on both ends & a circle ...



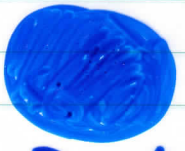
This process is called a "homotopy". Two spaces are "homotopic" if we can deform one into the other without cutting or pasting any parts of the object.

Lets look at a few sets of spaces & decide if they are homeomorphic or not:

1

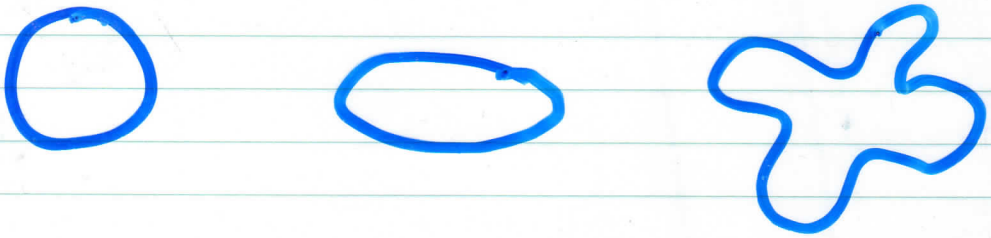


CIRCE



DISK

2



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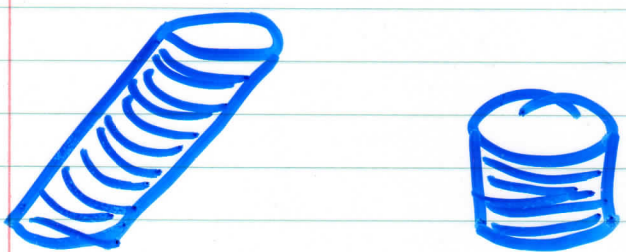
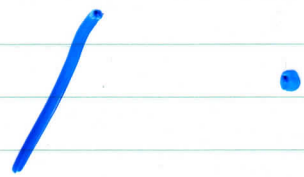
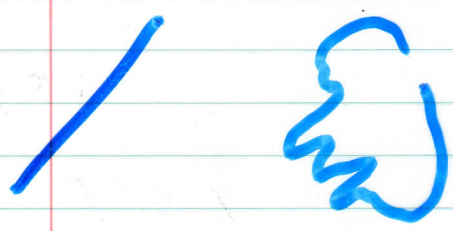
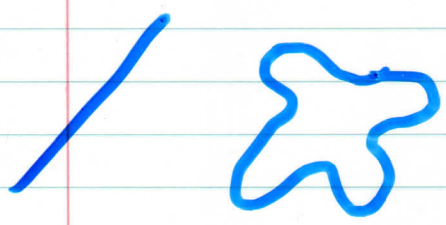
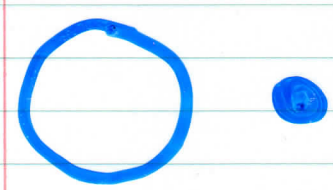


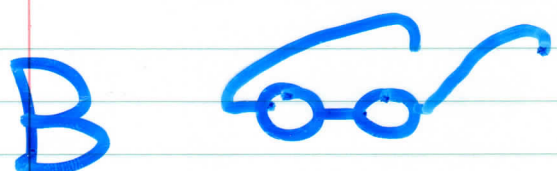
ex/ are the following parts homotopic! If so, show some steps to get from one to the other

a) 

b) 

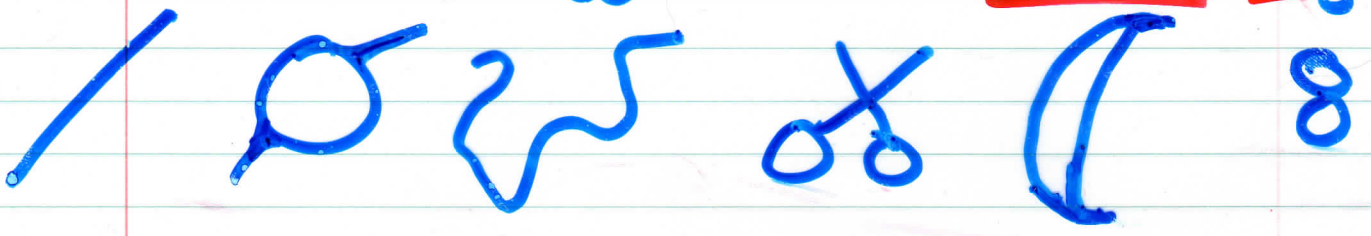
c) 

d) 

e) 

ex// Subdivide these objects into homotopy classes

line circle squiggle scissors banana eight



Section 6.2: Euler's Characteristic & 2-Manifolds

Consider our earth. Since it is a huge sphere & we are small, it may appear to us that it is flat as a plane.

So a sphere feels locally like the plane (but a sphere is bounded & the plane is not).

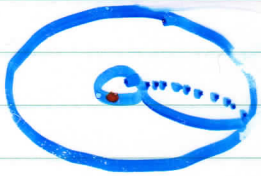
Such spaces (that feel locally like the plane) are called "2-manifolds".

? What is a 2-manifold other than a sphere ?

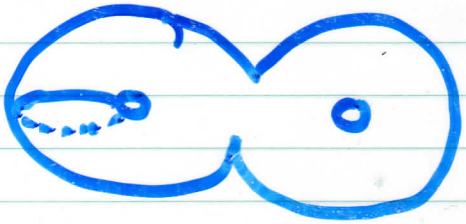
"2-MANIFOLDS"

Most basic 2-manifold: SPHERE

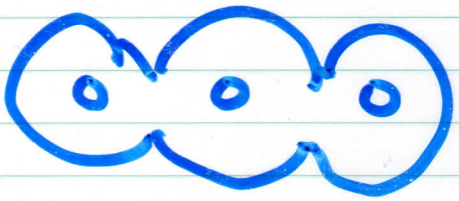
TORUS (donut):



CONNECTED SUM OF 2 TORI:

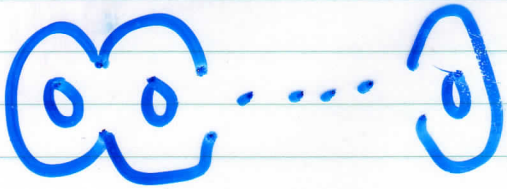


CON. SUM OF 3 TORI:



⋮

CON. SUM OF N TORI:



Any bounded space that feels like the plane will be homeotopic to one (& only one) of the spaces above.

Recall: The Euler's Characteristic (E.C.) is:

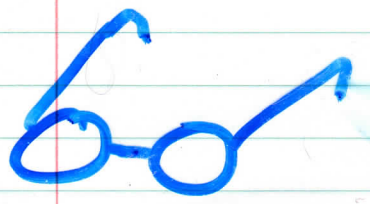
? Which 2-manifold are our platonic solids homotopic to?

An easier way to compute the E.C. is to relate it to an objects "genus".

Genus: the # of circular cuts you can make without separating the object

So genus & the Euler Characteristic uniquely specify the type of 2-manifold.

ex// Find genus, & E.C. for the objects below:

a) 

b)