CHAPTER 4: 3-D SPACES & RELATED OBJECTS

Section 4.1: Perspective

Though we think of our world as being three-dimensional, our basic visual perception is inherently two-dimensional. When we see, 2-D images are projected onto our retinas, & then the 3-dimensionality is decoded by our brain using the knowledge about associating distances, order, sizes, shadows, etc. that we acquire in our childhood.

When we attempt to draw the 3-D world we see we are essentially trying to project the images of these 3-D objects on the picture plane.

This process of projecting images of objects, is called "perspective".
from 3D to 2D

Basic Rules of Perspective Representation:

1. Objects far away from the "painting plane" are depicted as smaller compared to the same objects when they are close to the painting plane.

This rule may seem obvious to us, but we often take it for granted.
By consciously & cleverly violating this rule, we can produce visual paradoxes!

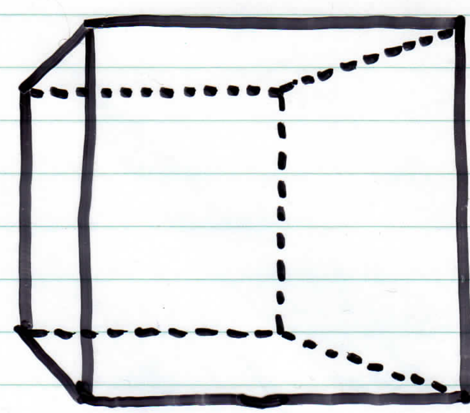
2. Lines that are parallel to the painting plane (p.p.) are depicted as parallel lines.

3. Mutually parallel lines that are NOT parallel to the p.p. are depicted as intersecting lines.

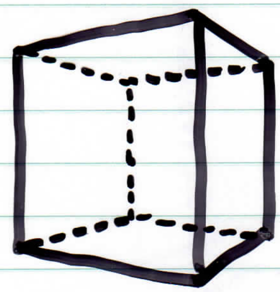
4. If we have 3 pairs of parallel lines such that the two lines in any of the pairs are mutually parallel & such that all of the lines are parallel to a fixed plane but not parallel to the picture plane. Then each of the pairs of mutually parallel lines are depicted as intersecting, & the 3 points of intersection lie on a single line.

Each cube (or "parallelopiped") contains 3 classes of mutually parallel edges, so there are 3 possible positions of the cube with respect to the picture plane:

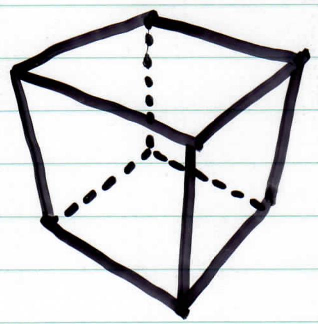
a) 2 of the 3 classes of parallel edges are parallel to the picture plane:



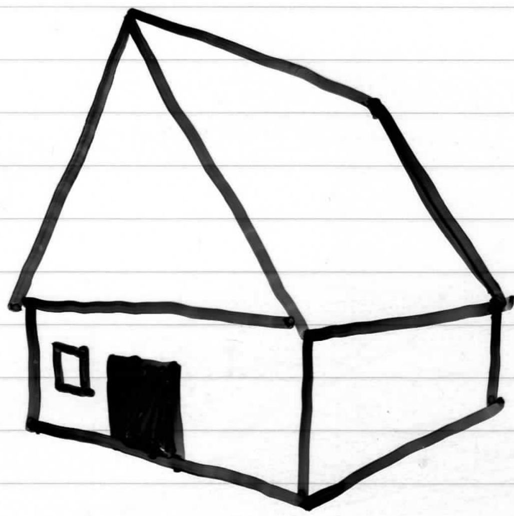
b) One class of parallel edges is parallel to the picture plane:



c) None of the edges is parallel to the picture plane:



ex// Find the horizon line in the following:



The idea of perspective drawing evolved in art. Early painters seemed to have some trouble with the idea of projecting 3-D images onto a plane.

Section 4.3: 3-D Objects; Conic Sections

Cones are important objects from a historical point of view.
During the 17th century (& earlier) they were the main route to studying certain important planar (2-D) curves.

Namely, circles, ellipses, parabolas, hyperbolas.

All of these curves can be obtained by intersecting one or 2 cones with suitably chosen planes. Hence the name "conic sections".

Lets see how we get the 4 conic sections.

1. Cut with a plane that intersects all of the slant heights of a cone:

2. Cut with a plane that intersects the cone & is parallel to some "tangent plane":

③ Cross-cut a double cone with any plane that intersects both halves:

We can use tangents (straight lines) to draw these (not so straight) shapes!

This is what we call "string art".

A) PARABOLA: ① Draw 2 line segments of equal length sharing a common vertex.

② Subdivide both line segments into an equal number (n) of smaller segments of equal length.

③ Label on one segment from the end (1→n), the other from the vertex (1→n).

④ Join each # to its corresponding pair.

B) HYPERBOLA: (this one does NOT use tangents)

① Start with 2 parallel lines (m & n) & one line that is perpendicular to both (l).

② Label the intersection points A & B, these are the two vertices of the hyperbola.

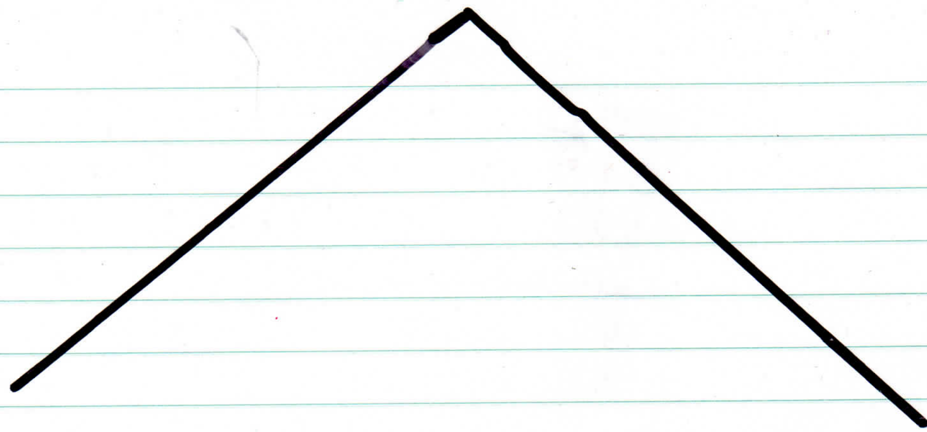
③ Identify the midpoint of AB; O.

④ Draw circles (any radius) centred at O.

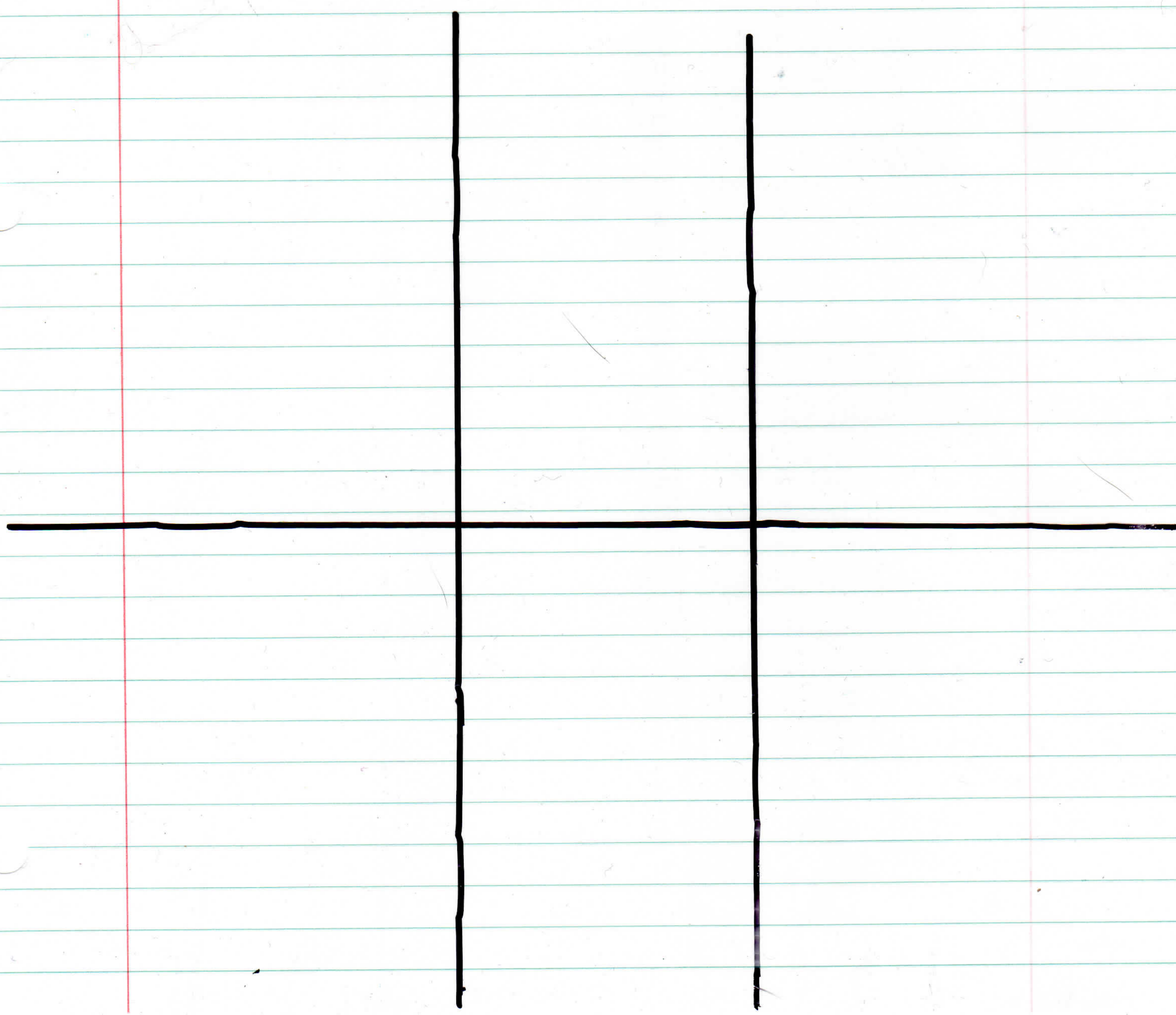
⑤ Draw horizontal lines through the points of intersection of the circles & m & n & vertical lines through the points of intersection of the circles with l.

⑥ The intersection points define the hyperbola!

A)



B)



C) ELLIPSE: (1) Start with a rectangle with a pair of horizontal & a pair of vertical sides.

(2) Then extend the vertical sides & mark the mid-point of each of the two vertical sides 0 & the top vertices by 1.

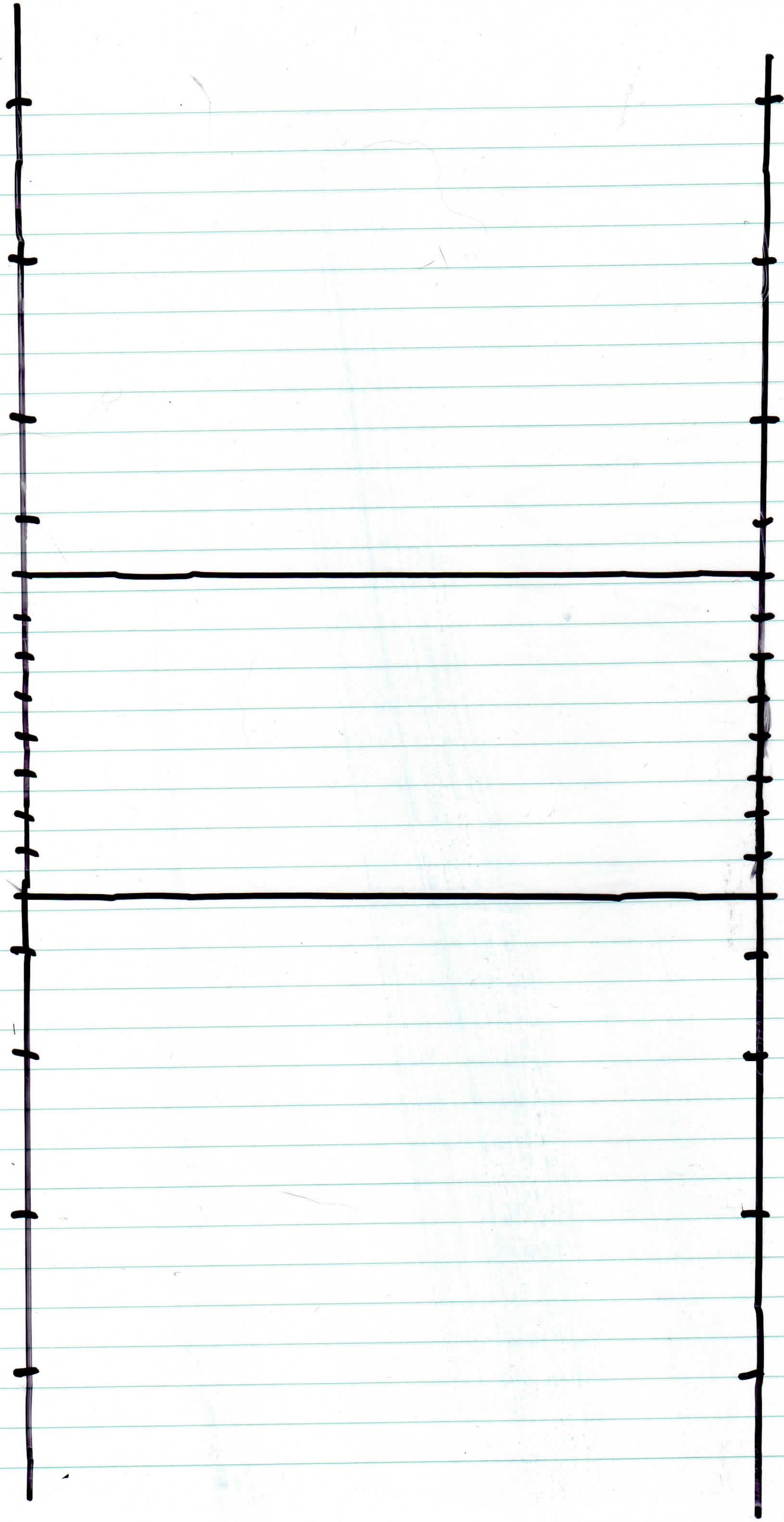
(3) Now any other point on this line is marked by the number that measures its distance to 0 using units we establish; + above 0, - below.

(4) Continue the labeling above the rectange (with the same units).

(5) Connect each point marked m on one side to the point marked 1/m on the other side.
reciprocal

(6) The resulting line segments will be tangent (skim) the ellipse & the more line segments, the clearer the outline!

c)



D) CARDIOID: (not a conic section)

1. Start with a circle.
2. Subdivide it into a number (n) of equally spaced points on the circle.
3. Label the points starting with 1 & ending with n.
4. Join a point to its double

5. Once we reach the limit L, join the next point to the remainder we get after we divide the number (2)L by the number of points n.

D2) We can do the same thing by drawing a circle & fixing a point A on that circle, then draw circles centred at various points of the starting circle & passing through A.