Section 2.4: Wallpaper Designs & Tilings of the Plane

Recall the frieze patterns. They had only one type of translational symmetry: along vectors that were integer multiples of one fixed vector.

This is why our patterns extended in rows.

? What happens if we have more translational symmetries?

In fact, any translation that is a symmetry of this design can be represented as a composition of translations along u or -u and v or -v.

An object in the plane is a "wallpaper design" if:

a) There are 2 non-parallel vectors (u & v) such that every translation that can be obtained by composing a number of translations along u or -u followed by a number of translations along v or -v is a symmetry of that object.

b) Every translational symmetry of the object MUST be of the above kind.

As was the case with frieze patterns, wallpaper designs have also been classified according to their symmetry groups.

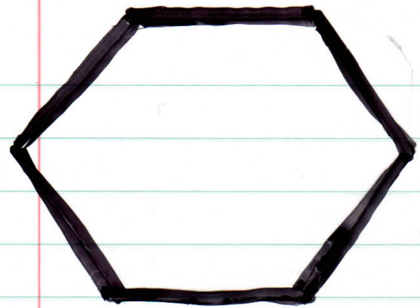
There are 17 symmetry groups of wall-paper designs.

Any arrangement of (planar) objects on a plane in such a way that all of the plane is covered is called a "tiling of the plane".

The objects used to cover the plane are "tiles".

triangles, squares, regular pentagons, ... n-gons to cover the plane we have a "regular polygonal" tiling & if we use just one type of a tile, we have a "regular monohedral" tiling.

Some math will help us analyze the possibilities...



Assume that we have a monohedral tiling of m-gons so that there are n many m-gons meeting at a vertex. All n of the interior angles at that vertex combine to make $360°$ →

There are 17 regular tilings with one or more than one type of tile.

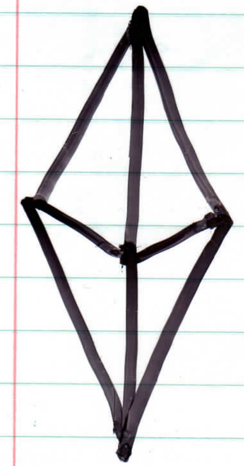
There are also tilings that do not have any translational symmetries - "aperiodic" tilings -

Section 4.2: Regular & Other Polyhedra

? How many regular polygons are there in 2-D?

In 3-D we have a "polyhedron", a bounded, 3-D solid object whose boundary is made up of (filled) polygons (called its "faces"). Also, throughout that boundary polygons have either common edges or common vertices.

We also want our 3-D objects to be COMPLETELY symmetric...



These "regular polyhedra" are called "Platonic Solids".

1) All bounding polygons are congruent regular polygons

2) Each vertex of a bounding polygon is adjacent to the same number of bounding polygons

? How many regular polyhedra are there?

Cube-

Tetrahedron-

Octahedron-

Dodecahedron-

Icosahedron-

If we let our 3-D solid now have faces of one or more types of regular polygons we have "semi-regular polyhedra".

? How many are there ?

If we exclude prisms, then there are only 13 semi-regular polyhedra that are not Platonic solids (or prisms). We call these "Archimedean Solids".

Back to Platonic Solids:

SOLID	F	E	V	E.C.
Tetrahedron				
Octahedron				
Dodecahedron				
Icosahedron				

This does NOT hold for all polyhedra.

eg// Calculate the E.C. for the below: