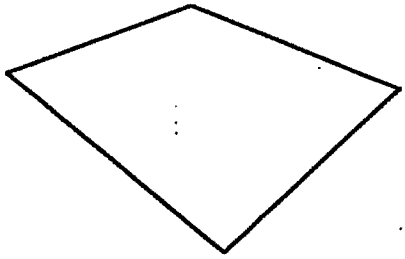
[9] **5.** The picture below depicts a square drawn in two-point perspective.

(a) Find the horizon and the vanishing points.

(b) Construct a 2 x 2 chessboard in the given square in the given perspective drawing. (That is, subdivide in the perspective drawing the given square into 4 equal smaller squares.)

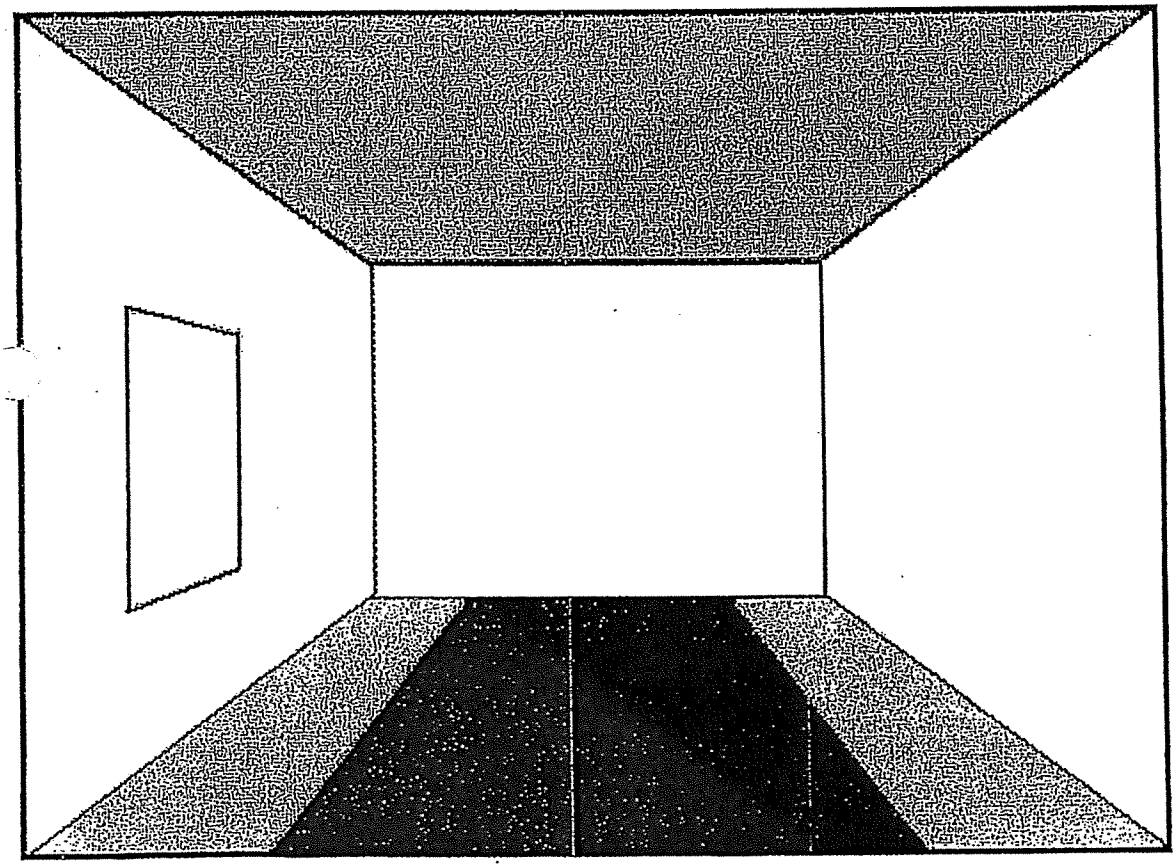


THE UNIVERSITY OF MANITOBA

Dec 13, 2005
Paper #221
Departments & Course Numbers: 136.102/054.02
Examination: Math in Art
Examiners: Dr. R. Padmanabhan and Prof. T. Lysenko

Final Examination
Time 2 Hours

6. The museum room given below is drawn in perspective.
Locate all the vanishing points in the diagram
Draw a table in the middle of the carpeted floor.
Add a window on the right side of the wall corresponding to the one on the left.
All the drawings should be drawn consistent with the given perspectivity.



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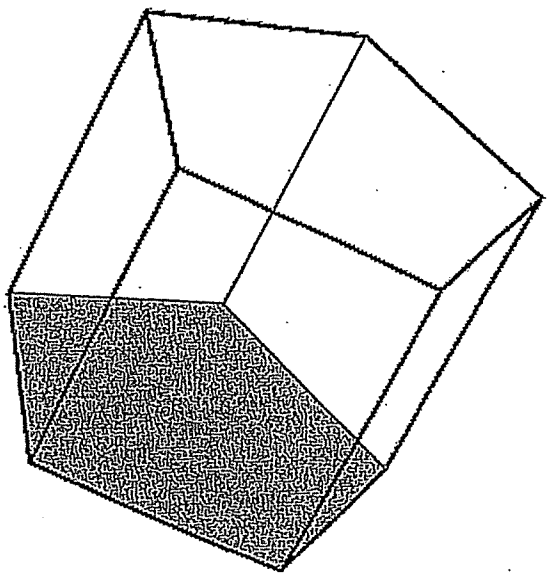
7. (a) Complete this table for the five Platonic solids.

Name of the solid	V = # of Vertices	E = # of Edges	F = # of Faces	# of Sides of each Face	# of Faces at each Vertex	V-E+F
Tetrahedron	4	6	4	3	3	2
Cube						
Octahedron						
Icosahedron						
Dodecahedron						

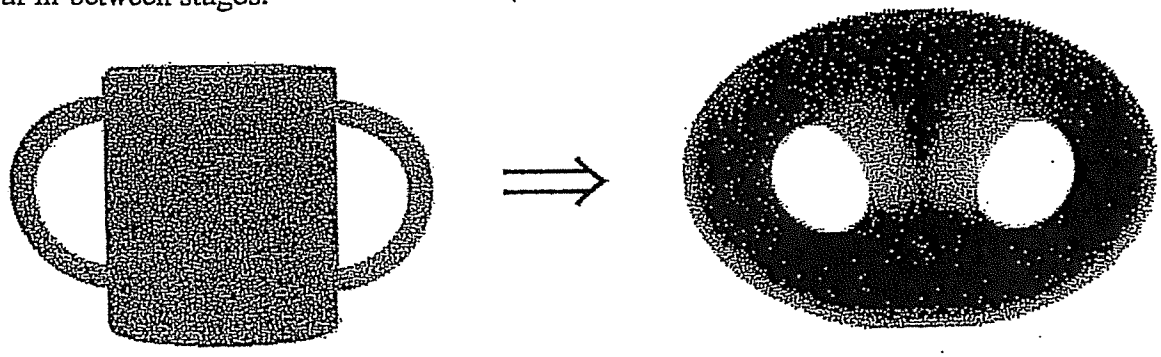
7 (b) State the Euler's theorem giving a formula connecting the vertices, edges and faces of a convex polyhedra.

(c) Calculate the values of V, E and F for the polyhedron shown below.

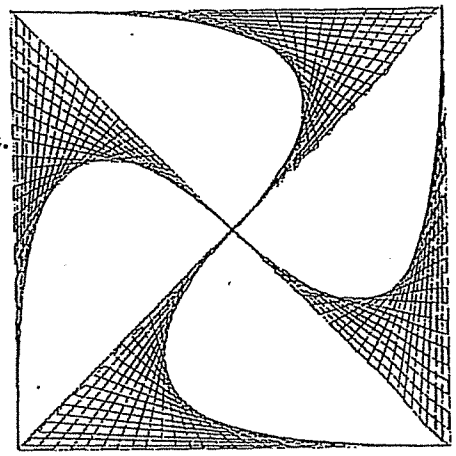
(d) What is the value of V–E+F here?



6 Apply a sequence of topological transformations (e.g. the continous moulding of, say the ceramics or clay), to obtain the shape of a face mask shown on the right from the shape of the coffee mug with two handles as shown on the left. Show at least three or four in-between stages.



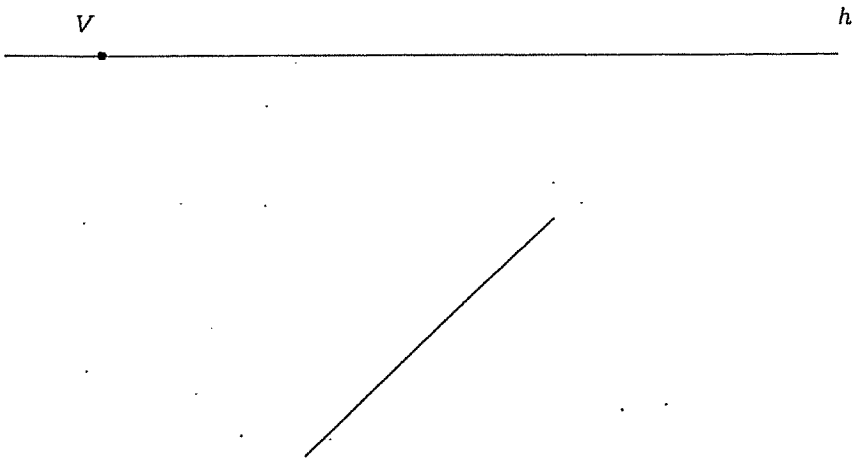
4. Using the technique of drawing straight lines as tangents, draw four smooth parabolas in the square given below to create a shape as shown on the right side.



[9] 6. Below is depicted a portion of a drawing of a square in 2-point perspective. One side of the square is given, along with one of the vanishing points V (not the vanishing point associated with the given side), and the horizon line h.

(a) Indicate the 2^{nd} vanishing point with symbol V_2. Construct the other three sides of the perspective square.

(b) Divide the square into 4 smaller squares (i.e. a 2×2 grid).



7. You are given (see the picture) the horizon line, two vanishing points VP1 and VP2 and two **opposite** corners A and B of a square (in perspective drawing).

(a) Construct a two point perspective drawing of the (rest of the) square (with A and B being its opposite corners).

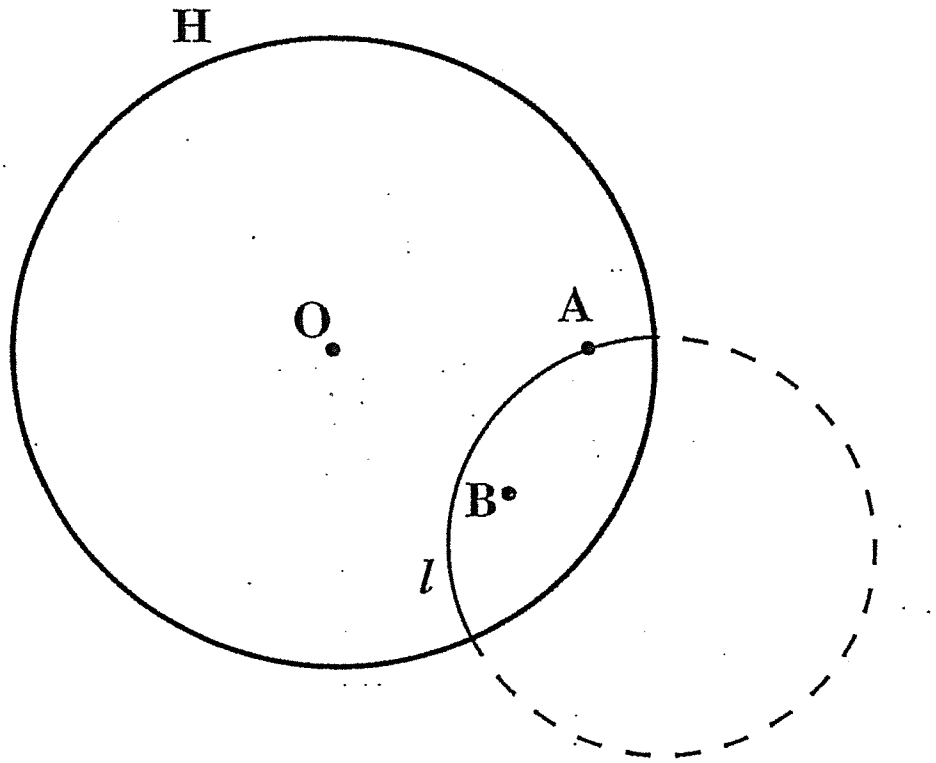
(b) Subdivide (tile) the square constructed in (a) into 4 equal smaller squares (in the perspective drawing done in part (a)).

5 + 5 = 10

[8] **7.** We are given a hyperbolic line *l*, a point *A* on that hyperbolic line, and a point *B* outside the line *l*.

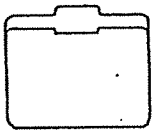
(a) Construct one hyperbolic line (label it h_1) parallel to *l* and passing through *B*, and one hyperbolic line (label it h_2) intersecting *l* and passing through *B*.

(b) Construct the hyperbolic line passing through both *A* and *B*.

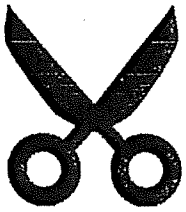


[7] 8. The objects depicted below consist of the black coloured points only.

(a) Which of the following four designs are mutually homotopic?

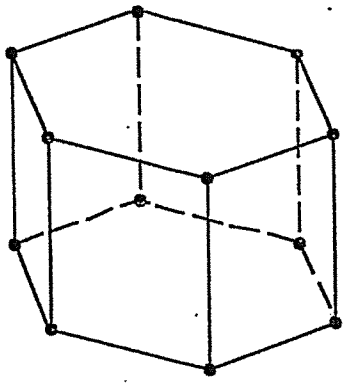
(1)  (2)  (3)  (4) 

(b) Show that the two designs shown below are homotopic by drawing at least three in-between sketches showing the left object can be continuously deformed into the right object.

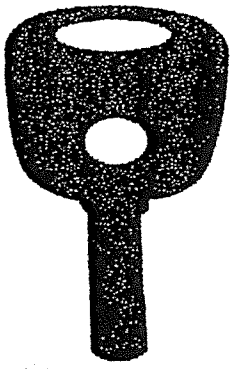
1. (a) State the Euler's formula connecting V-the number of vertices, E-the number of edges and F-the number of faces of a convex polyhedron (no proof is required).

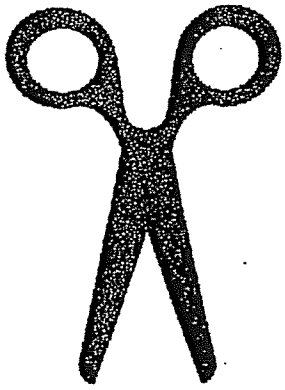
(b) Find the numbers V, E and F for the hexagonal prism given below. Calculate the value of V - E + F for this convex polyhedron.



(c) If a convex polyhedron has 10 vertices and 7 faces, find the number of edges of the polyhedron. Draw such an example of a polyhedron in the space given below.

2. Demonstrate that the two images given below are homotopic by drawing at least three in-between images showing how the key can be continuously deformed into the scissors.





7. (a) Give the Euler's formula connecting V, E and F for a convex polyhedron

(recall V = # of verticies, E = # of edges and F = # of faces).

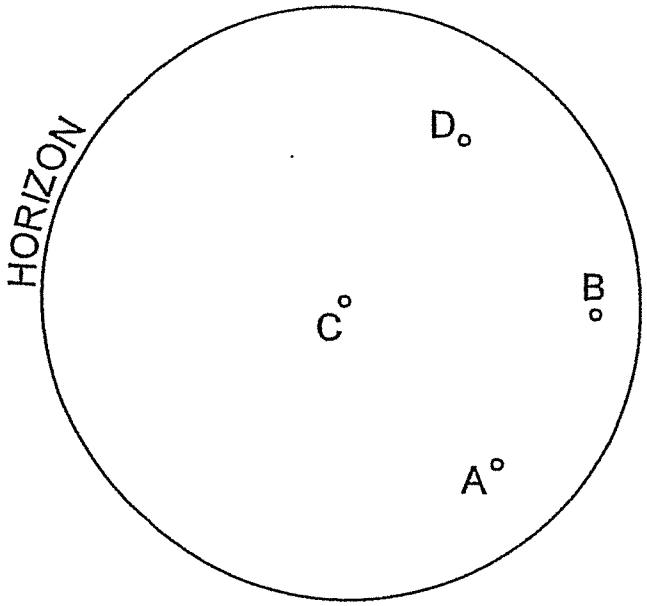
(b) Let P be the convex polyhedron obtained by glueing together two identical regular tetrahedra along one face (to obtain a triangular dipyramid). Find the values of V, E and F for this polyhedron. Calculate the value of V–E+F.

(c) Is this a regular polyhedron? Give reasons for your answer.

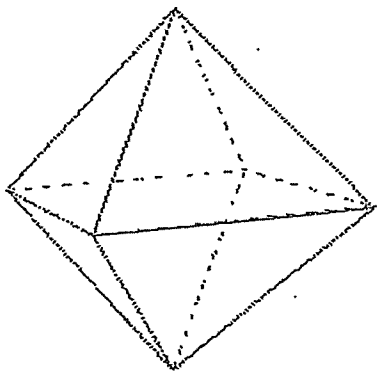
8. Hyperbolic Plane Constructions.

(a) Draw the hyperbolic line joining the two points C and D.
(b) Draw the hyperbolic line joining the two points A and B.
(c) Draw <u>two</u> hyperbolic lines through the point D and parallel to AB.
(d) How many such lines can be drawn through D and parallel to AB?

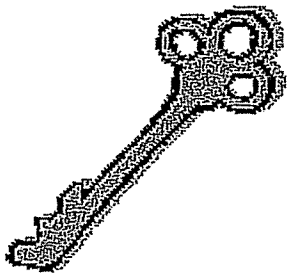
Note that the point C is the centre of the horizon circle.



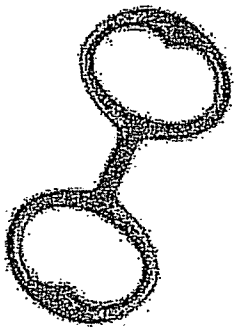
9. (a) Compute the Euler characteristic of the octahedron (see the picture).

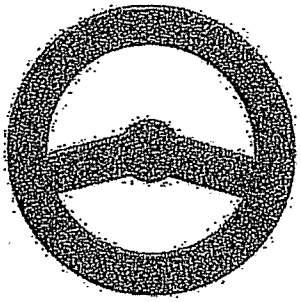


(b) What is the genus of the **surface** of the key depicted below? (Note that the key is not flat – it is a 3 dimensional object).



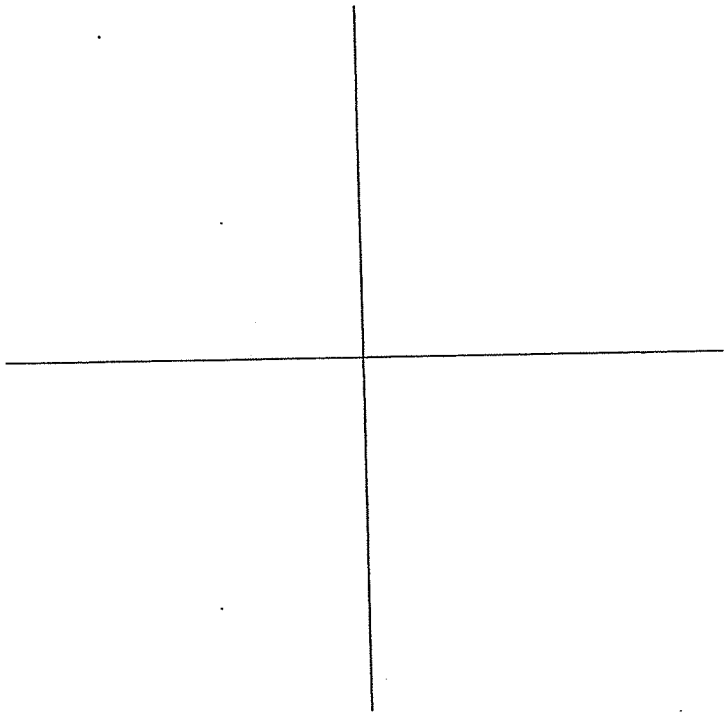
(c) Show that the two objects given below are homotopically (topologically) equivalent by drawing at least 3 in-between sketches showing how one can be continuously deformed into the other.





4 + 2 + 4 = 10

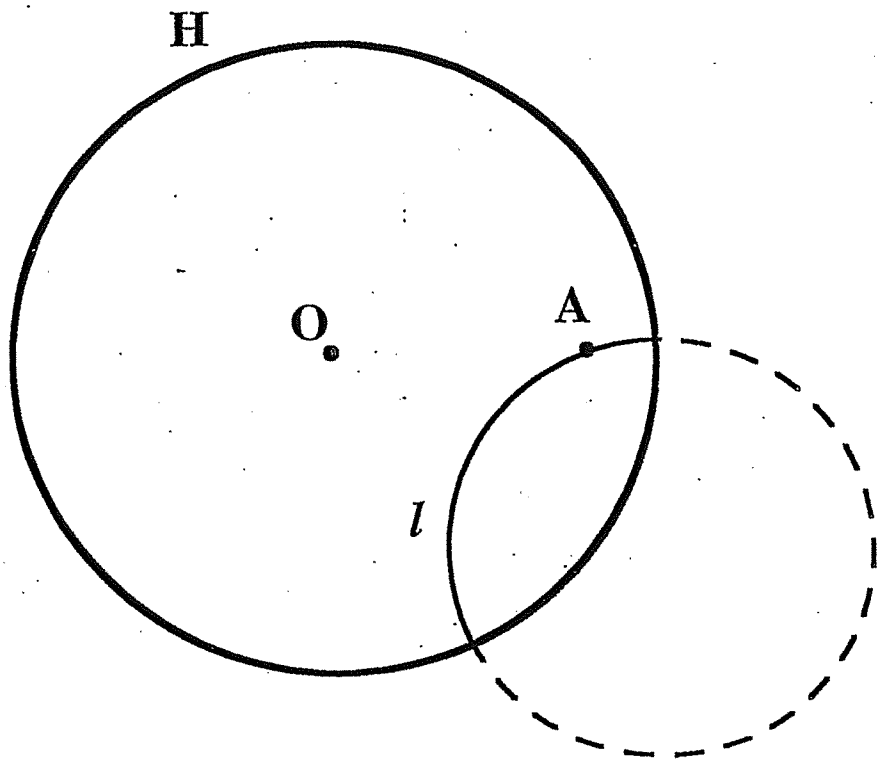
[7] 5. On the following axes, form a star by constructing the tangents a parabola in each corner. Each parabola should have 4 tangents. (You may wish to use the space at the bottom to find the appropriate divisions of the line. It is not necessary, however it will make your work neater.)



[10] **7.** We are given a hyperbolic line l and a point A on that hyperbolic line.

(a) Construct one hyperbolic line parallel to l.

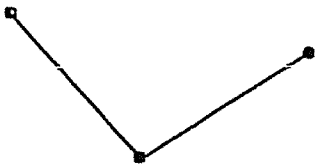
(b) Construct the hyperbolic line passing through A and perpendicular to l.



7. The illustration below shows the horizon h and a two-point perspective drawing of two adjacent edges of a square.

(a) Find the two vanishing points, and construct in perspective the remaining two edges of the square.
(b) Subdivide the perspective image of the square in part (a) into four equal smaller squares.

h



[7] 8. (a) Which of the letters D, S, B and P are homotopic?

(b) Show that the letters A and R are homotopic by drawing at least three in-between sketches showing how A can be continuously deformed into R.

A R

[8] 4. (a) Fill in the following table with the appropriate information about the Platonic solids.

Name of Solid	number of Faces	number of Edges	number of Vertices
	12		20
cube		12	
icosahedron		30	
tetrahedron			4
	8	12	

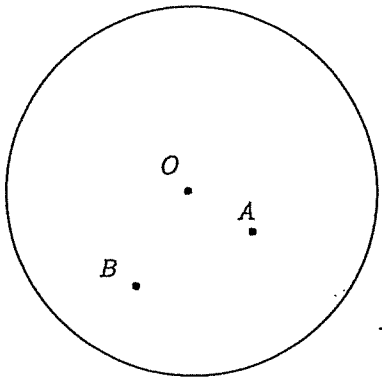
(b) The icosododecahedron is an Archimedean solid that has 12 pentagon faces and 20 triangle faces. How many edges and how many vertices does a icosododecahedron have?

(Hint: there is enough information given to calculate the number of edges. Use the Euler characteristic to find the number of vertices)

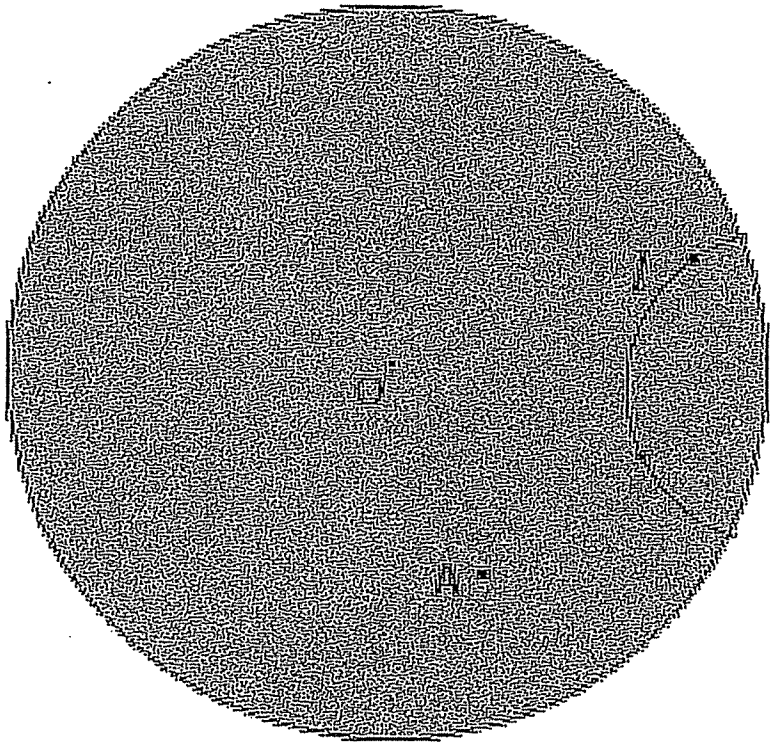
[9] · 7. Below is a hyperbolic plane, having center 0.

(a) Construct the hyperbolic line, labeled h_1, that passes through the points A and B.

(b) Construct one hyperbolic line that intersects h_1, label it h_2, and one hyperbolic line that is parallel to h_1, label it h_3.

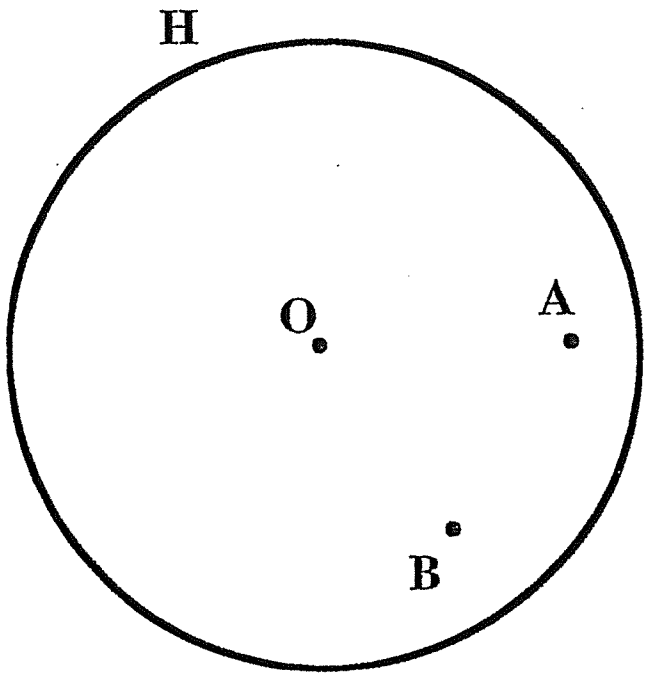


8. We are given a hyperbolic line *l* and a point A in the Poincare model of the hyperbolic geometry (please see the picture below).



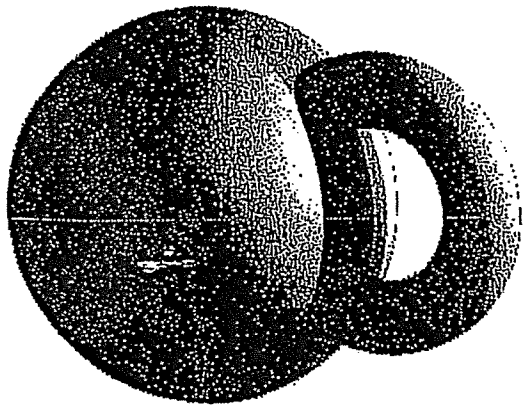
Construct two hyperbolic lines through the point A and parallel to the given hyperbolic line *l*. Leave the intermediate constructions (do not erase them); **briefly** explain your steps.

7. Construct the hyperbolic triangle with vertices A, B and O as shown below, where O is the center of the horizon circle H.



8. (a) Compute the genus of the three-dimensional **surface** depicted below (a sphere with a handle).

(b) Find the Euler characteristic of the object shown below; use part (a) to justify your answer.



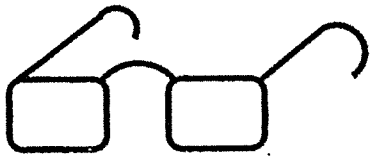
[8] 8. (a) Identify which letters and numbers are mutually homotopic in the following phrase:

BEN PARK ROOM 368

(b) Since I have 20\20 vision, I own a pair of glasses that have no lenses (depicted below). Note, they are just frames. We want to consider just the surface of these glasses frame.

What is the genus of the surface of these glasses frame?

What is the Euler characteristic of the surface of these glasses frame?



Frames of a pair of glasses
(A 3 dimensional object)

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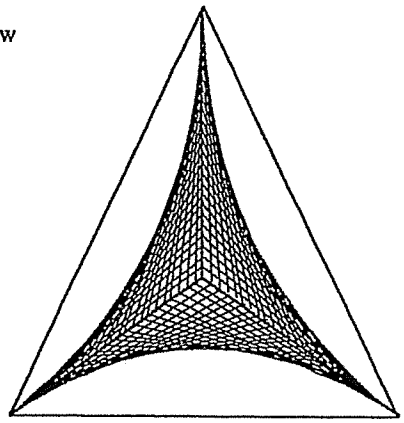
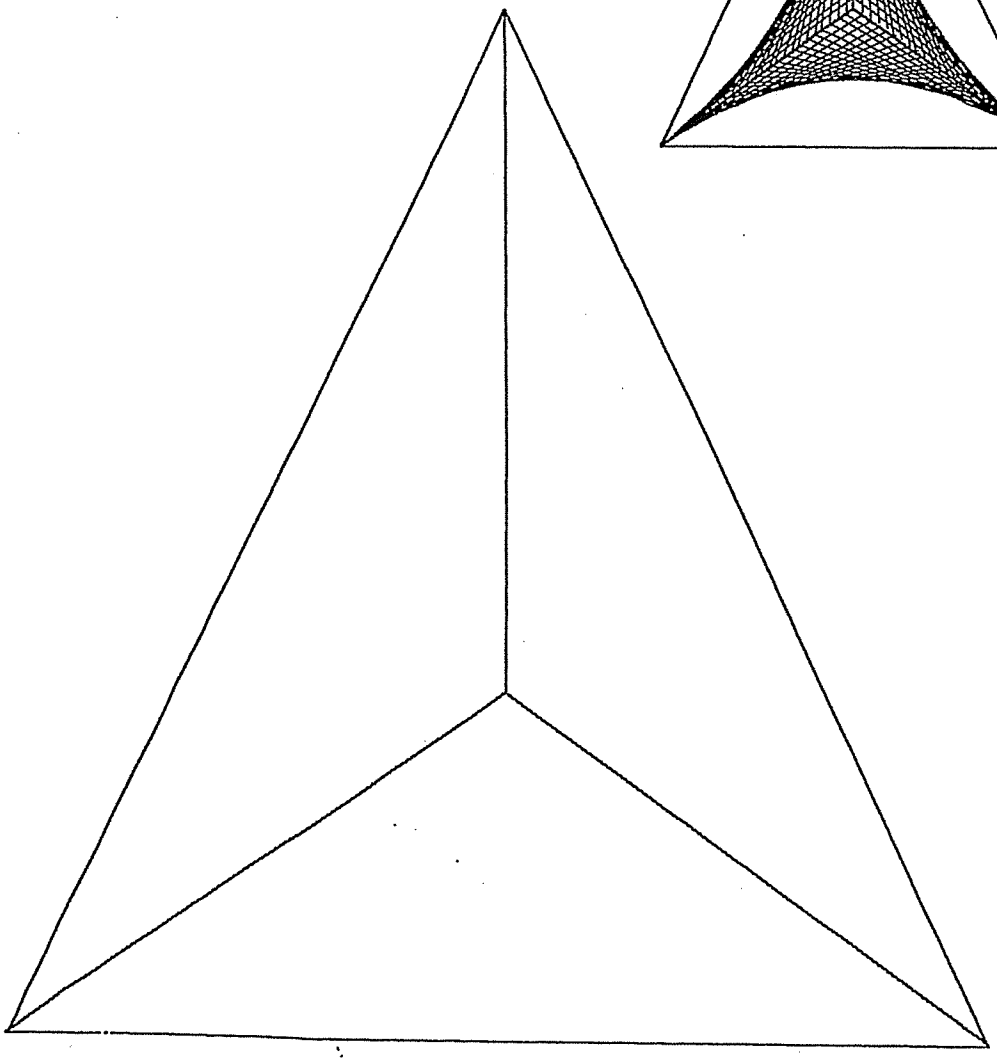
8. Using the technique of drawing straight lines as tangents, draw three parabolas in the space given below to create a shape as shown on the right side.



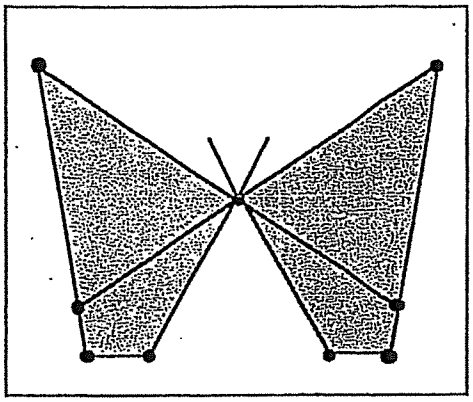


#8

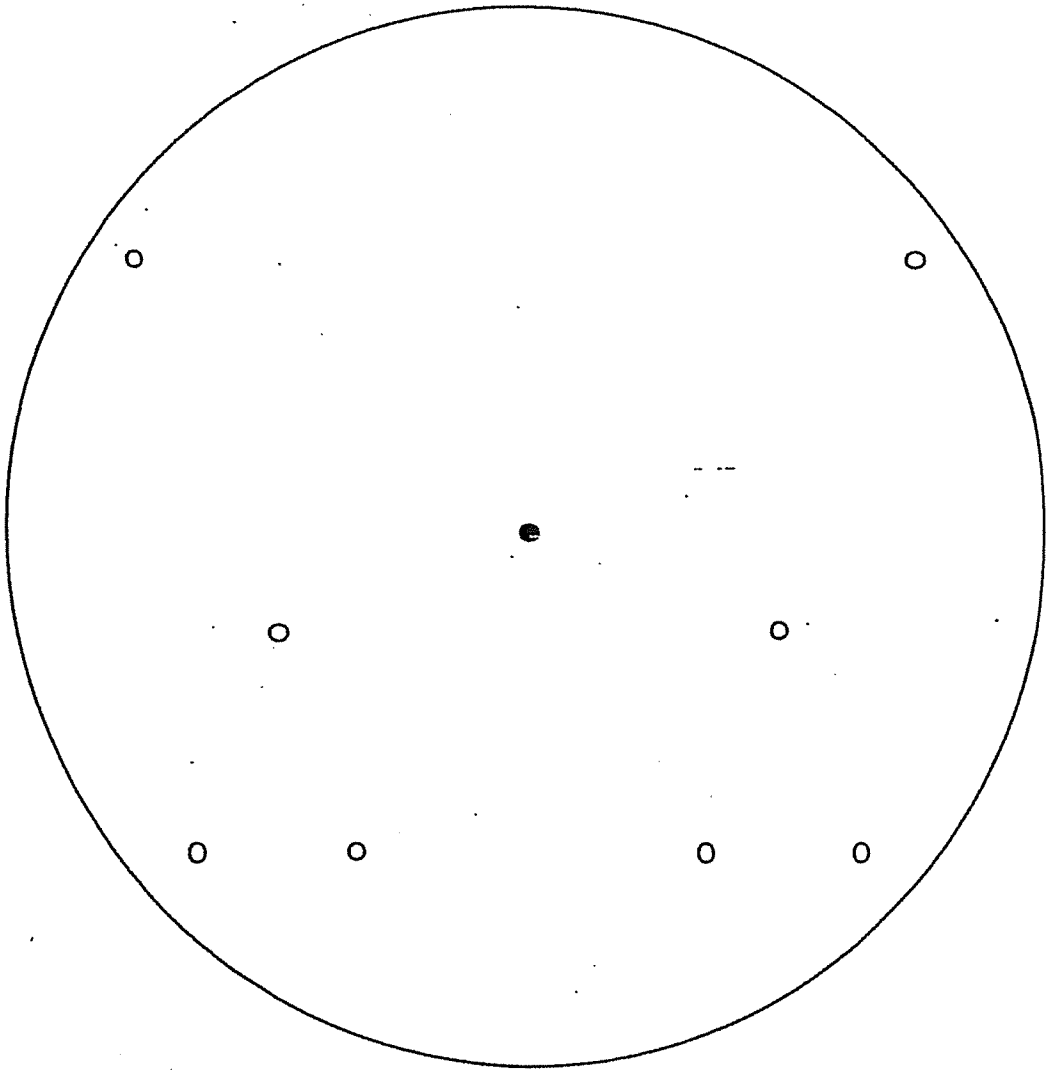
On the right hand side you are given a line drawing of a stylistic artwork of a butterfly drawn in the usual Euclidean plane. Reproduce this art on thehyperbolic canvas given below.

In your new drawing, the euclidean lines will be redrawn as hyperbolic straight lines.

(Here the point O is the centre of the hyperbolic canvas)



stylistic butterfly in Euclidean canvas



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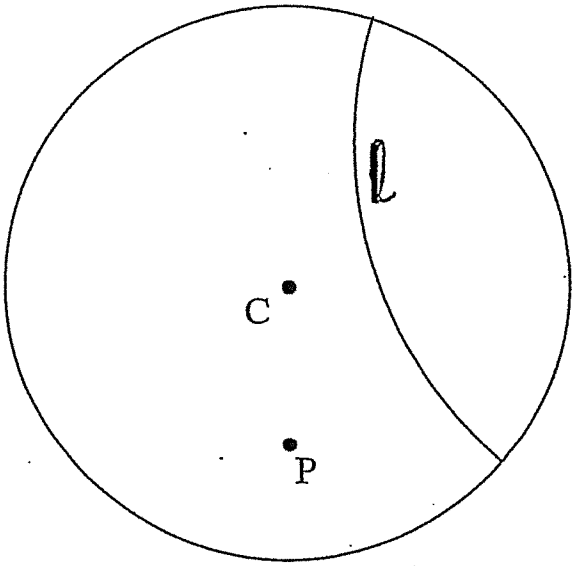
4.In the figure given below, the circular disk is the Poincaré model of the hyperbolic canvas.Here ℓ is a hyperbolic straightline and P is a point not on ℓ.Draw two hyperbolic straight lines passing through P and parallel to ℓ. How many such straight lines are there in this geometry?



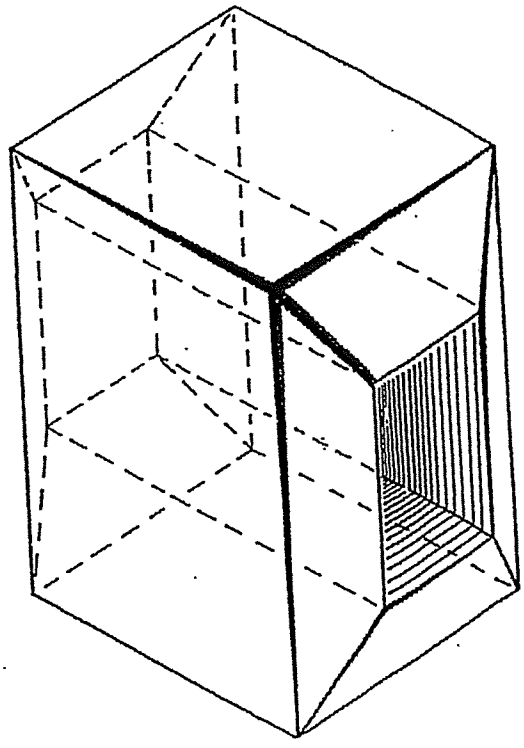
#5

Complete this table for the five Platonic solids.

Name of the solid	V = # of Vertices	E = # of Edges	F = # of Faces	shape of each Face	# of Faces at each Vertex	V-E+F
Tetrahedron						
Cube						
Octahedron	6	12	8	triangle	4	2
Icosahedron						
Dodecahedron						

State the Euler's theorem connecting the vertices, edges and faces of a convex polyhedra.

Determine the values of V, E and F for the polyhedron shown here.
Find the value of V – E + F.
Does this contradict the Euler's Theorem? Give reasons for your answer.



3. Construct a **hyperbolic** square with vertices at A, B, C, and D. [You may use the fact that the four vertices are equidistant from the center O of the Poincaré model we are using, and that the distances between every two consecutive vertices of the square are the same.]