Math 1020 Math in Art
Midterm Exam, February 24, 2009
BRIEF SOLUTIONS OR EXPLANATIONS

Name: _____ **Student Number:** _____

1.	*max=10*	
2.	*max=10*	
3.	*max=6*	
4.	*max=14*	
5.	*max=10*	

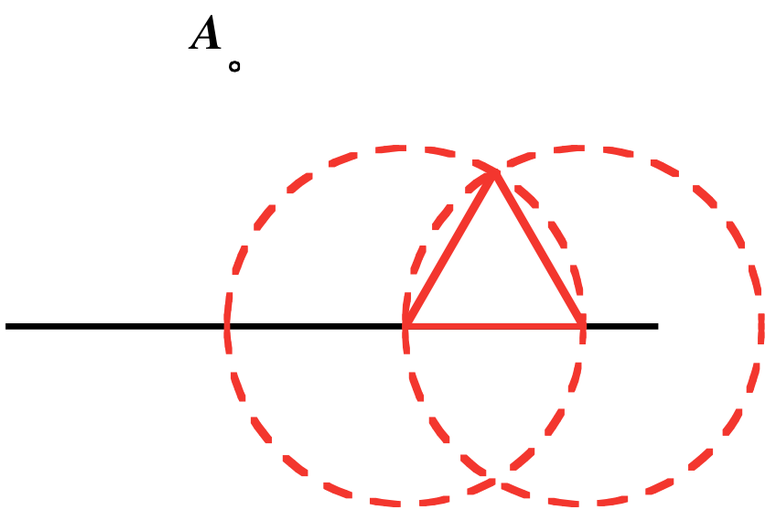
Total=50

Important: **"Construct" means "construct using an unmarked ruler and a compass". The phrase "unmarked ruler" stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do NOT erase them). Use words to describe BRIEFLY what you have done.**

[10 points] 1. (a) Construct the line passing through the given point *A* and that is parallel to the given line *l* (see the picture below).

Solution: This has been done many times in class, and it is also in the book.

(b) Construct one line passing through the point *A* and intersecting the given line at an angle of 60°.



Solution.
Construct an equilateral triangle as shown in red. Then construct a lime parallel to one of the two sides of the red triangle and passing through the point A (as in part (a) of this question.)

[10 points] **2.** **(a)** Construct a golden rectangle over the line segment *AB* (so, the line segment *AB* should be the largest side of the golden rectangle).

(b) Construct a golden spiral within the golden rectangle you have constructed in part (a).

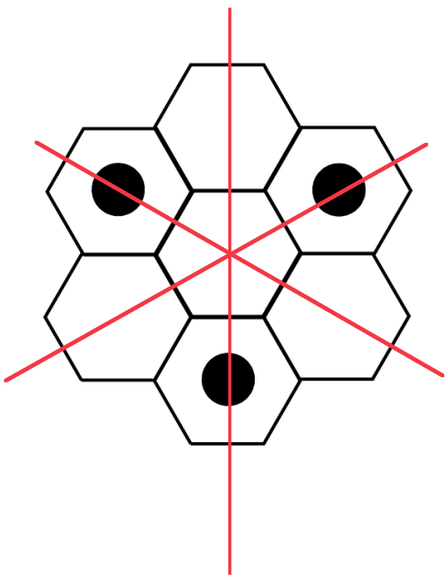
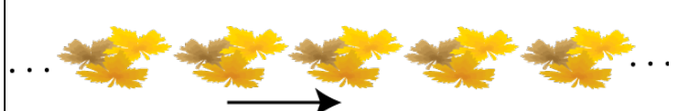
Solution. First find the golden cut C of AB such that C is closer to B. The height of the golden rectangle is the same as CB. The rest was shown in the notes and in the textbook.

[6 points] **3.** **(a)** State precisely the definition of the sequence of Fibonacci numbers.

(b) The 19th Fibonacci number is 4181, and the 22nd Fibonacci number is 17711. Find the 20th and the 21st Fibonacci numbers.

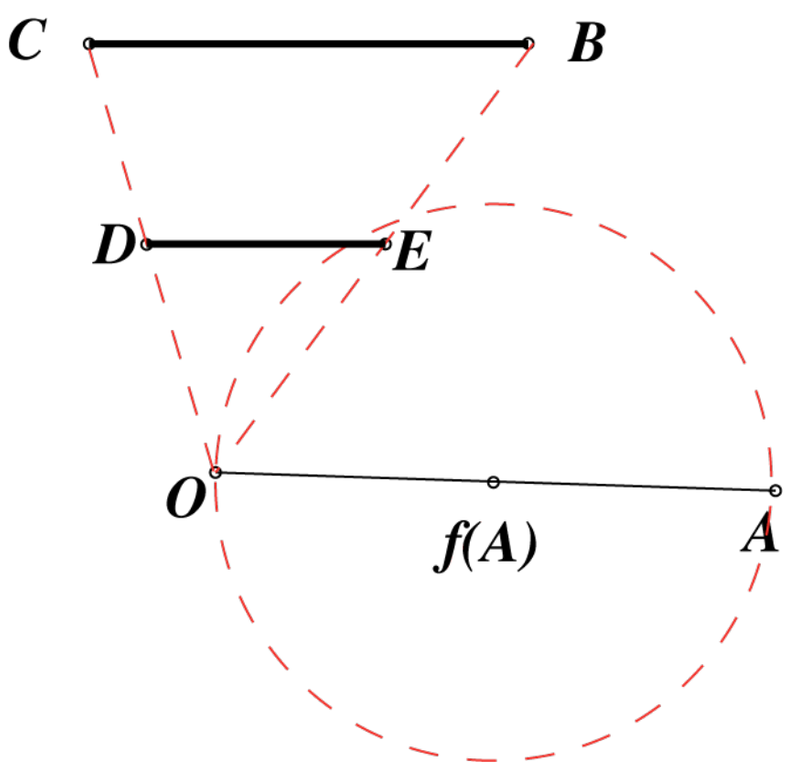
Solution. The sequence f_n of Fibonacci numbers is defined by $f_1 = 1$, $f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$.
In part (b) we are given that $f_{19} = 4181$ and that $f_{22} = 17711$. We want to find $x = f_{20}$ and $y = f_{21}$. It follows from the above definition that $4181 + x = y$ and $x + y = 17711$. We solve this by putting $4181 + x = y$ for *y* in the second equation: we get $x + 4181 + x = 17711$, and solving for *x* yields $x = 6765$. So, $y = 10946$.

[14 points] **4.** Find the group of symmetries for each of the three objects shown below. If you claim a rotational symmetry, indicate the center of the rotation and the angle of rotation. If there are reflections, show the line of reflection. If there are translational symmetries show or describe the vectors of translation, drawing **precisely** at least one of them. [In all three cases the object is defined by the (black or gray) coloured points.]

OBECT	THE GROUP OF SYMMETRIES
	Put a dot at the center of the design and call it O. The group of symmetries is {id, rot(O,90º), rot(O,180º), rot(O,270º)}
	Call the three red line m, l, and n, and call their common intersection point O. The group of symmetries in this case is {reflection with respect to l, reflection with respect to m, reflection with respect to n, identity, rot(O,120º) and rot(O,240º)}
 [This is a Frieze pattern and it extends without end both to the left and to the right.]	Denote the vector shown by **v** (you were expected to precisely draw that vector). The group of symmetries is $\{id, tran_{v}, tran_{2v}, \ldots$ $tran_{-v}, tran_{-2v}, \ldots\}$

[10 points] 5. We know that the point $f(A)$ is obtained from the point A by applying to it a central similarity f of stretching factor $\alpha = \frac{1}{2}$.

(a) Construct the center O of the central similarity f.
(b) Construct the image of the line segment CB under the central similarity f.



Solution. First find the point O as indicated. That would do part (a). Then find the midpoint *E of OB* and the midpoint *D* of OC. Join D and E, and done.