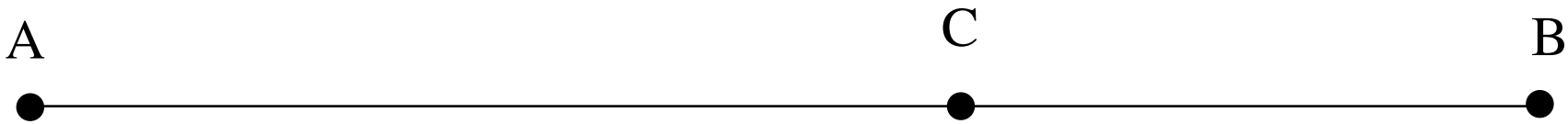
The Golden Ratio

Given a line segment AB, the point C on the line such that the ratio of the length of the line AB to the length AC is the same as the ratio of the length of line segment AC to the length CB.

This ratio is known as the golden ratio and is denoted by the greek letter ϕ. $\frac{AB}{AC} = \frac{AC}{CB} = \phi$

The Golden Ratio: $\frac{AB}{AC} = \frac{AC}{CB} = \phi$



Golden Ratio: Solutions to quadratic equations

An equation of the form:

$$ax^2 + bx + c = 0$$

has solutions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The value of the Golden Ratio

The value of the Golden Ratio

$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$$

(ϕ is the only positive solution to the equation $x^2 - x - 1 = 0$.)

Construction 1: The Golden Cut

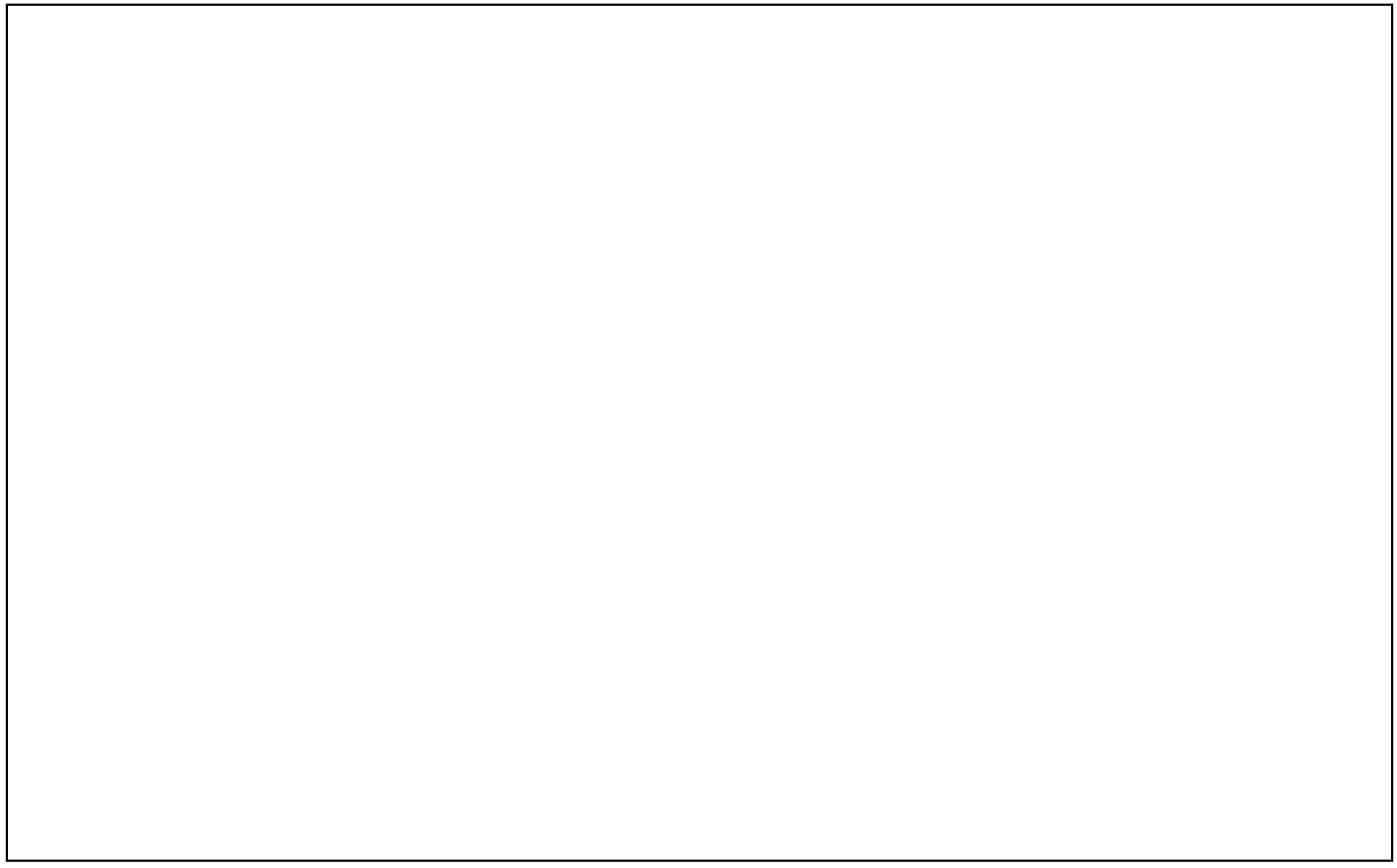
Golden Rectangle

A Golden rectangle is a rectangle that has side lengths that are in golden proportion.

$$\frac{b}{a} = \phi$$

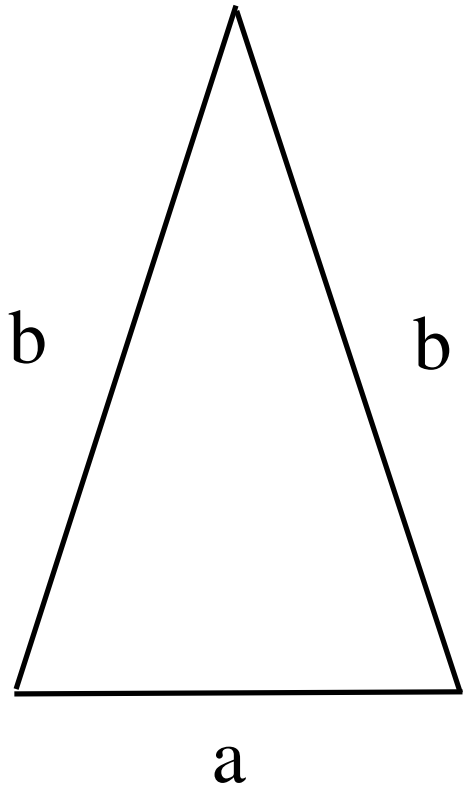
Construction 2: Golden Rectangle given shorter side

Construction 3: Golden Spriral

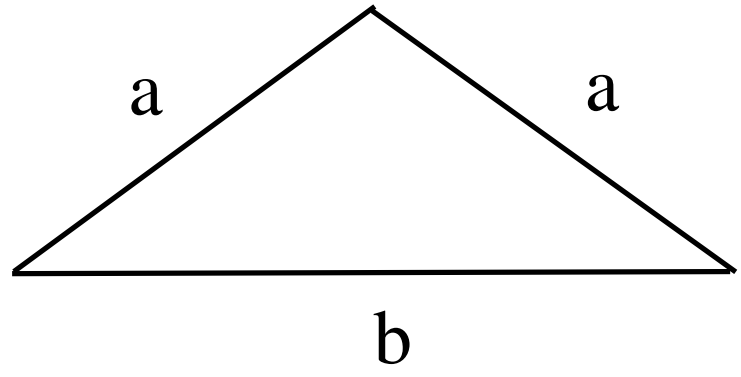


Golden Triangles

Golden Acute Triangle

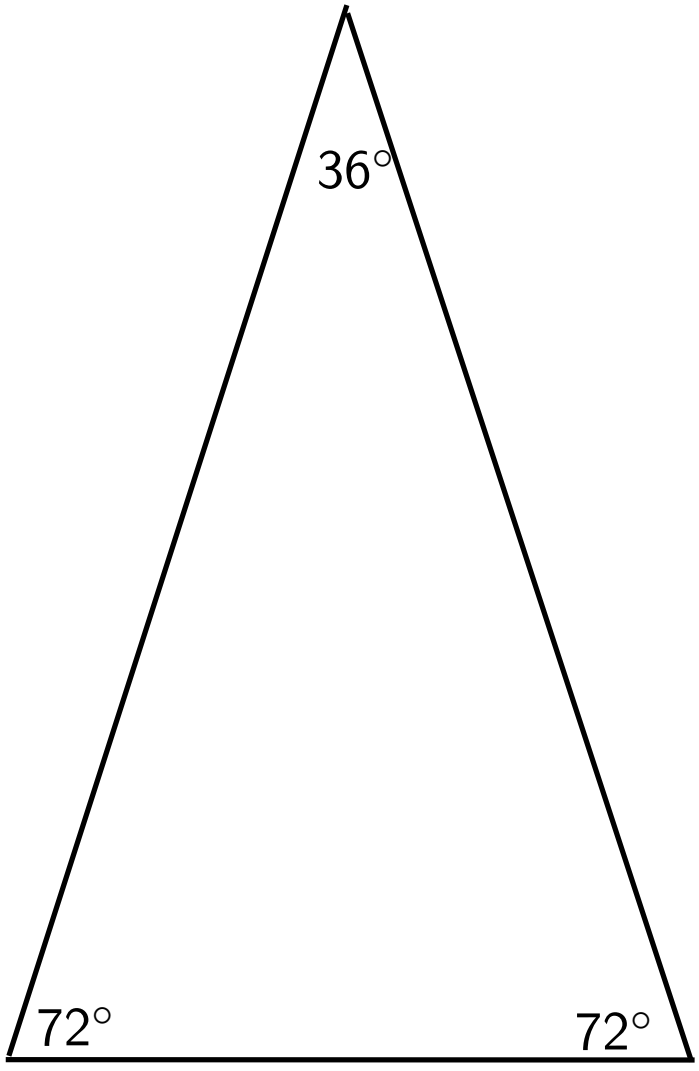


Golden Obtuse Triangle



$$\frac{b}{a} = \phi$$

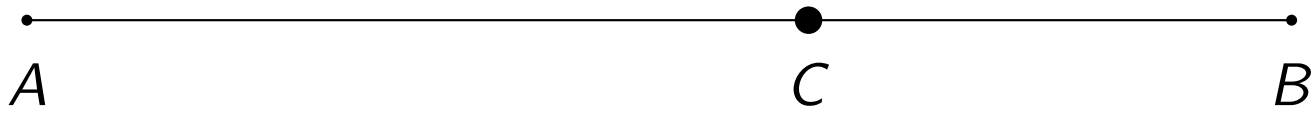
Is this a Golden Acute Triangle?



Construction 4: Acute Golden Triangle over a given base

Construction 5: Golden Triangles over a given base alternate construction

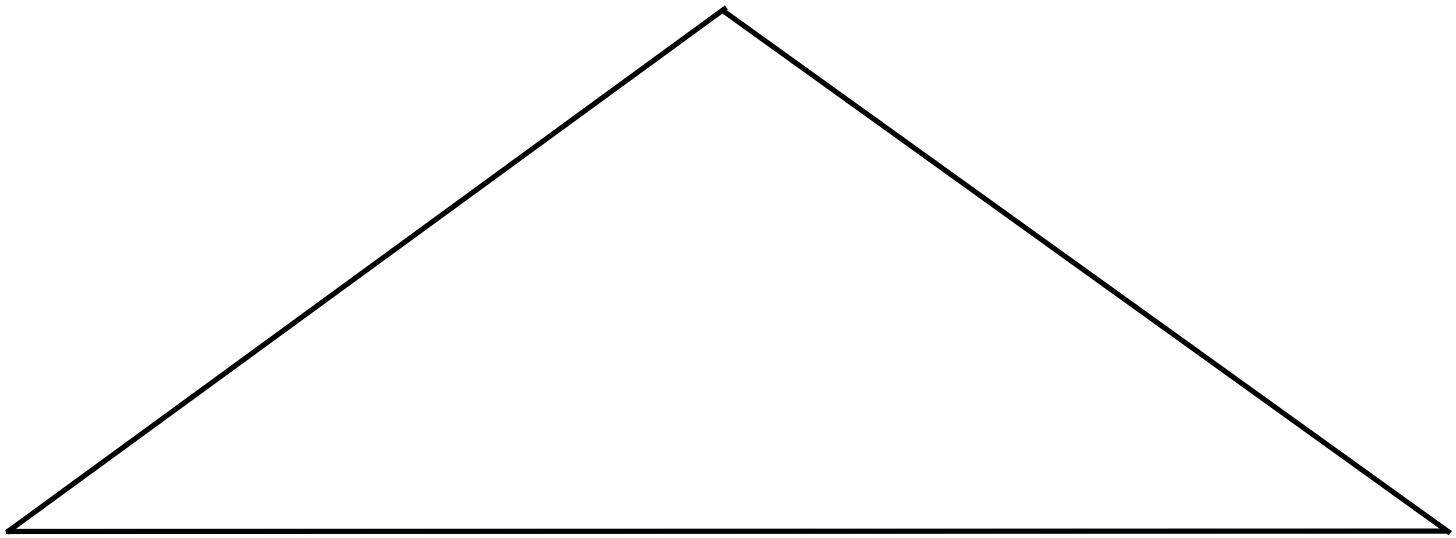
Suppose that a golden cut was done previously, and so you had lengths as in the line below.



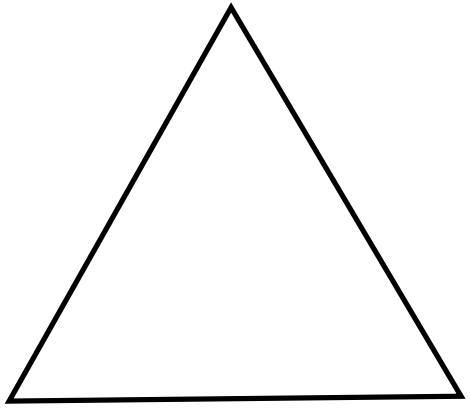
Obtuse

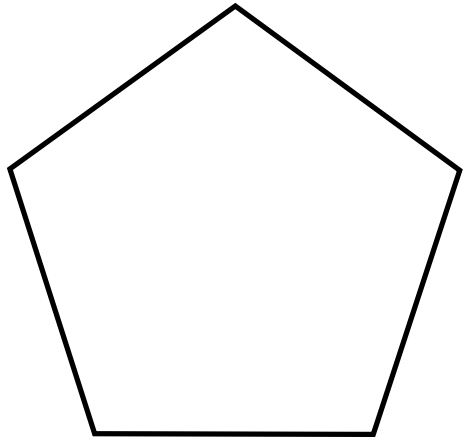
Acute

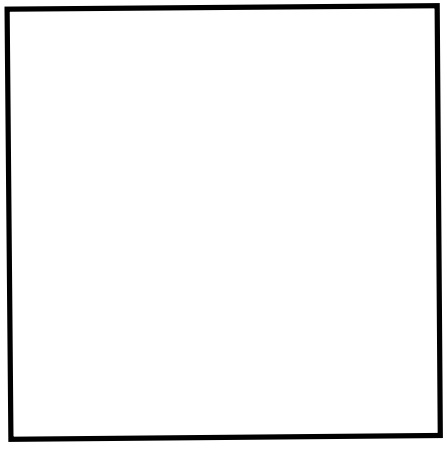
Construction 6: Subdividing an Obtuse Golden Triangle

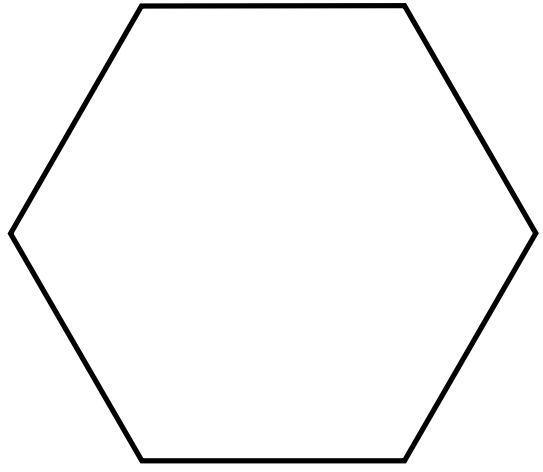


Regular Polygons









Construction 7: Regular Pentagon over a given side

Fibonacci Numbers: The Classic Problem (page 25)

A certain man puts a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair, which from the second month become productive?

Fibonacci rabbits, the first few months

1^{st} month

2^{nd} month

3^{rd} month

4^{th} month

5^{th} month

Fibonacci Numbers

Let f_n be the number of pairs of rabbits after n months.

Fibonacci Numbers

The Fibonacci Numbers are the numbers in the sequence defined by

$$f_1 = 1 \quad f_2 = 1$$

$$f_n = f_{n-1} + f_{n-2}$$

Binet's formula for Fibonacci numbers

$$f_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}$$

Given that $f_{19} = 4181$ and $f_{16} = 987$,
what are f_{17} and f_{18}?

Ratios of Fibonacci Numbers

$$\frac{f_2}{f_1} =$$

$$\frac{f_3}{f_2} =$$

$$\frac{f_4}{f_3} =$$

$$\frac{f_5}{f_4} =$$

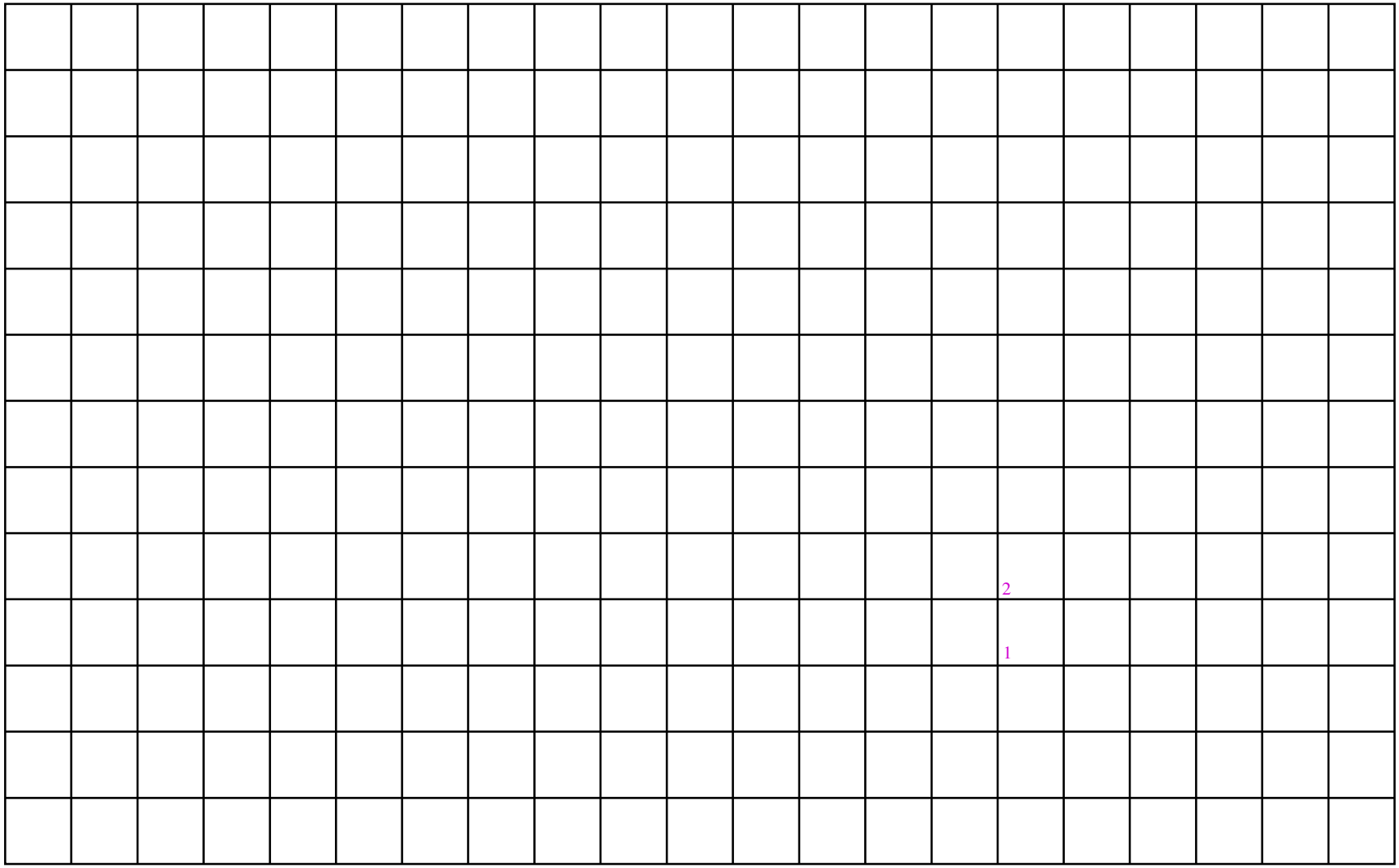
$$\frac{f_6}{f_5} =$$

$$\frac{f_7}{f_6} =$$

$$\frac{f_8}{f_7} =$$

$$\frac{f_9}{f_8} =$$

Fibonacci Spiral



Fibonacci Flowers