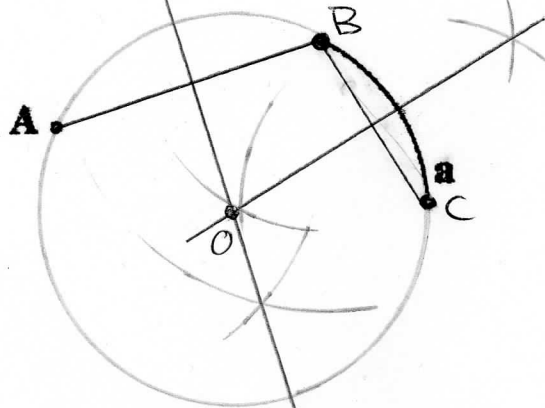


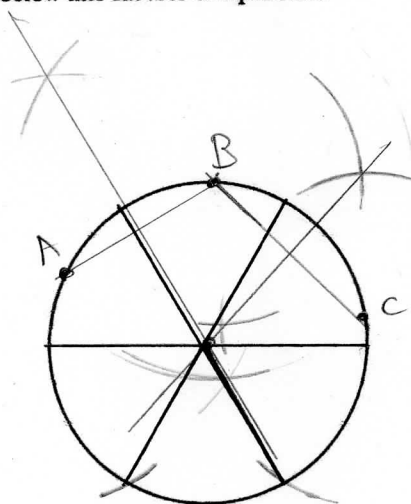
Show your work. Leave clear sketches of the intermediate steps you make in the construction. Clarity in your pictures will be appreciated and may affect the marking. Accompany your constructions with short descriptions of the steps you make.

1 [8]. (a) The arc a and the point A are all parts of an invisible circle. Construct (using an unmarked ruler and a compass) the center of that circle and then draw the rest of the circle.



- ① pick any 2 points on A (B & C)
- ② bisect AB & BC to find the center, O

(b) Subdivide the circle shown below into six arcs of equal size.



- ① find center
- ② build equil. triangles to split into 6 60° angles!

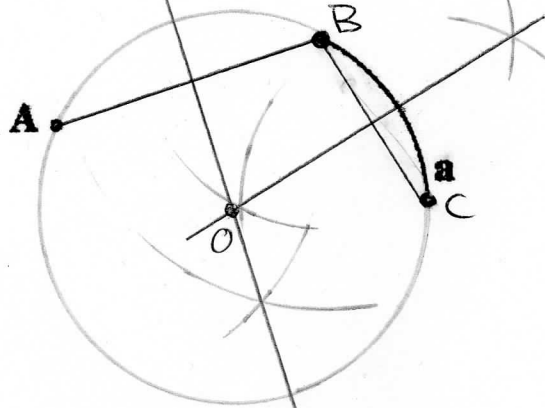
$\frac{360^\circ}{6} \rightarrow$ whole circle

6

$= 60^\circ$

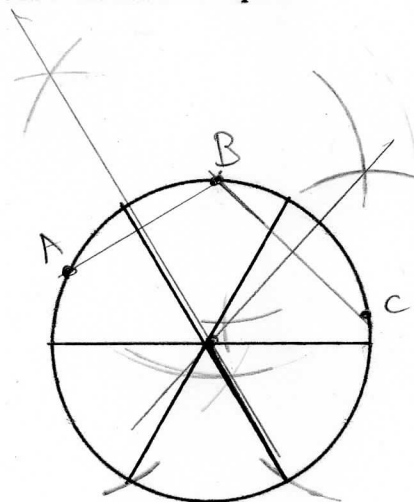
Show your work. Leave clear sketches of the intermediate steps you make in the construction. Clarity in your pictures will be appreciated and may affect the marking. Accompany your constructions with short descriptions of the steps you make.

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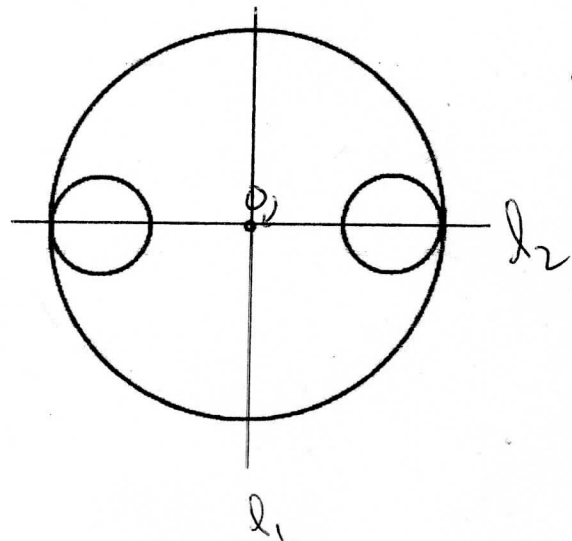
6

$= 60^\circ$

2.[9] Find the symmetries of the following objects.

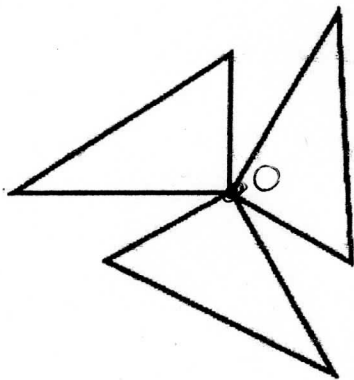
[If you list a rotation as a symmetry then identify the center and the angle of rotation. If you list a reflection as a symmetry then identify the line of reflection.]

(a) The small circle in the middle indicates the center of the large circle.



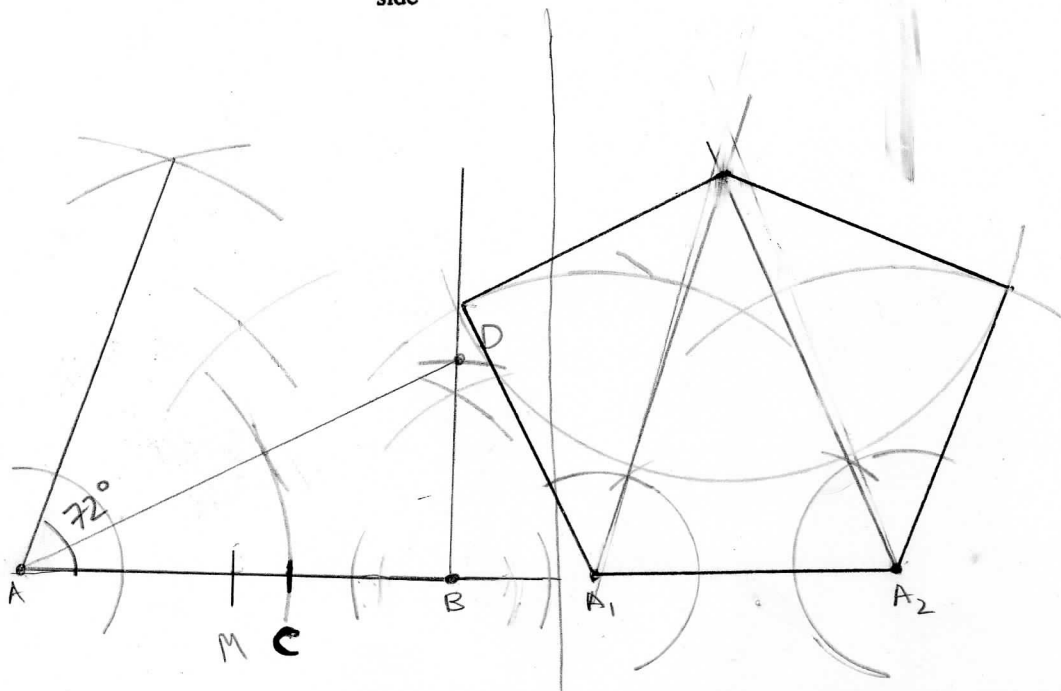
$\{I, \text{ reflection about } l_1, l_2,$
 $\text{ rotation about } O \text{ by}$
 $180^\circ\}$

(b)



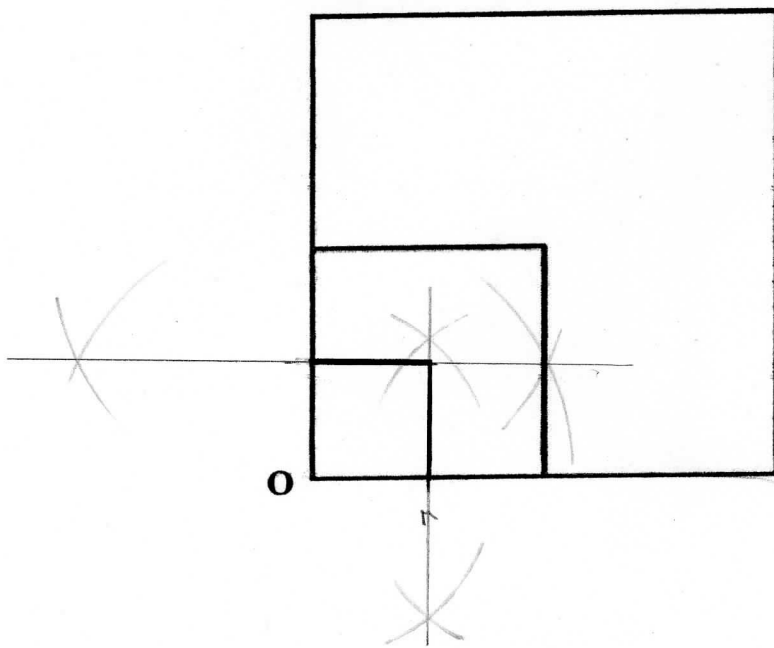
$\{I, \text{ rotation about } O$
 $\text{ by } 120^\circ, 240^\circ\}$

3. [8] (a) Construct an acute golden triangle over the given side (see below).
 (b) Use the golden triangle in (a) to construct a regular pentagon over the same side



- ① find the golden cut & make an acute g.t. elsewhere
- ② duplicate the 72° angle at A_1 & A_2 to create the acute g.t.
- ③ measure A_1A_2 & repeat at tips to make the pentagon

4. [8] The picture below shows a part of a fractal F. The following is known about F: the rest of the fractal is within the smaller square and the fractal F is self-similar with respect to the similarity centered at the corner point O and with stretching factor $1/2$. Draw (and explain in one sentence) the rest of the fractal F.



The rest of the fractal is continuing smaller squares created by a central similarity with $\alpha = 1/2$ & centred at O.

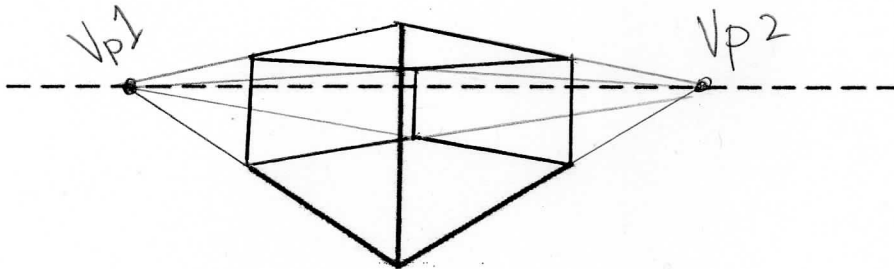
5.[9] (a) What kind of a polyhedron is a platonic solid? (Explain in one sentence.)

How many types of platonic solid are there? 5

symmetric

It is a polyhedra made of faces of regular polygons, all of one type!

(b) A cube is one type of a platonic solid. The following picture depicts the edges in one corner of a cube and the (dashed) horizon line of the perspective drawing. Draw the rest of the cube.



6. [10]

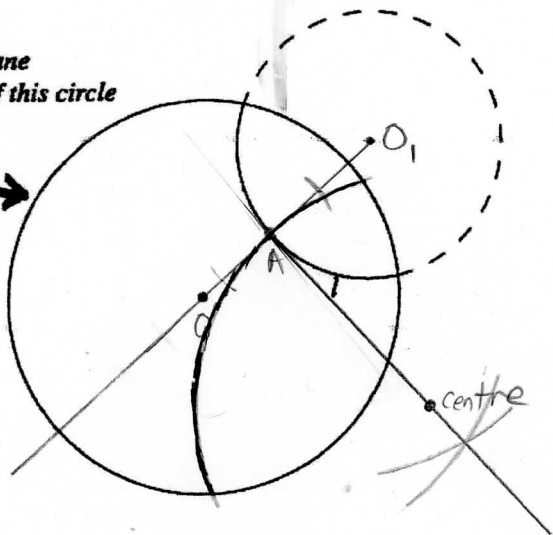
(a) Construct one (any) hyperbolic line that is perpendicular to the given hyperbolic line l .

① connect O_1 to O
to find A

② make a perpendicular
line at A

③ any circle with its ~~radius~~ centre
along this line that intersects
the given circle at A will
be perpendicular.

The hyperbolic plane
is in the interior of this circle



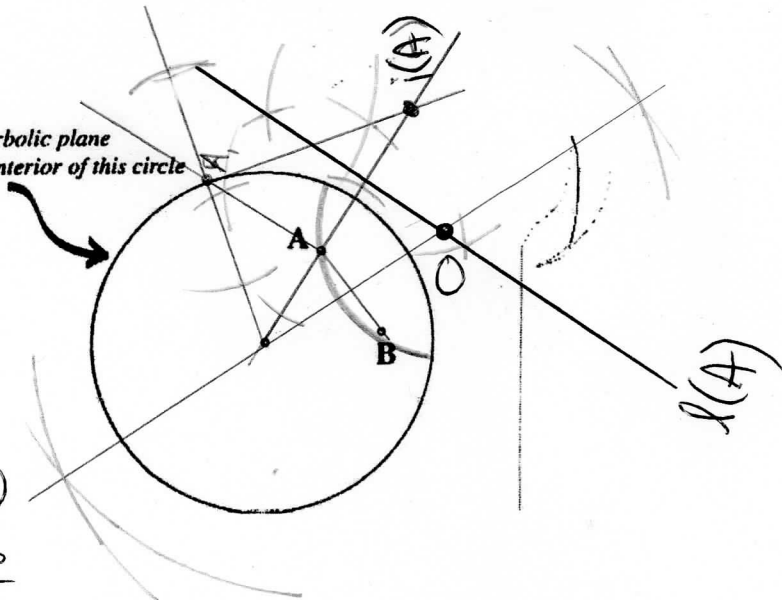
(b) Construct the hyperbolic line passing through the given points A and B.

① find $i(A)$
& $l(A)$

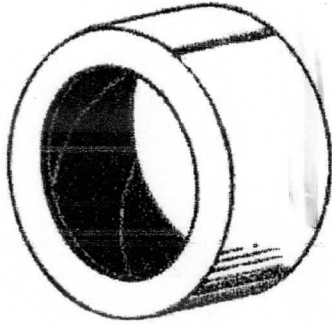
② bisect AB

③ the intersection
of this line with
 $l(A)$ is the center O
of the hyperbolic line
through A & B .

The hyperbolic plane
is in the interior of this circle



7.[8] Consider the surface of the following tape (the surface of the tape only).



(a) Find the genus of the surface of the tape. Briefly justify your answer.

genus = 1 \rightarrow 1 hole, or, can make 1 circular cut without disconnecting

(b) Use (a) to find the Euler's characteristic of the surface of the tape.

$$EC = 2 - 2(G)$$

$$\Rightarrow EC = 2 - 2(1) = 0 //$$

8. [5 bonus] True or false? (No need to justify your answer.)

(a) Every golden rectangle can be subdivided into exactly two squares. **F**

(b) The limit of the ratio $\frac{f_n}{f_{n+1}}$ is the golden section ϕ .

$$\text{F} \left(\frac{f_{n+1}}{f_n} = \phi \right)$$

(c) A composition of any two reflections with respect to a single line is the identity. **T**

(d) If f is a similarity of stretching factor 2 and centered at a point A, then the Julia set associated to f is the point A. **OMI**

(e) Every two bounded and orientable surfaces (= 2-manifolds) of the same Euler characteristic are homotopic. **T**