Topology; Homotopy

Two spaces are HOMOTOPIC if we can continuously deform one of them into the other without *cutting* or *pasting*.

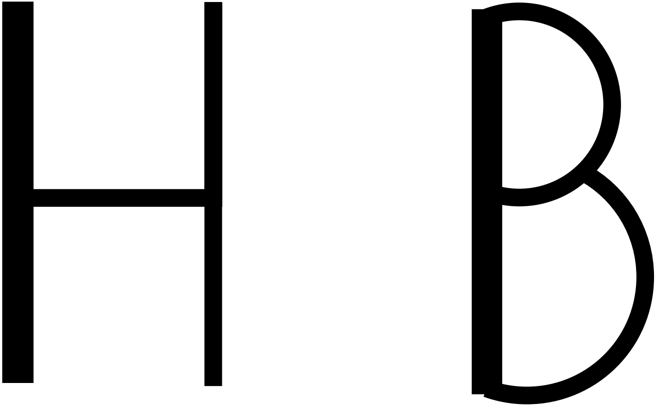
This deformation is called a homotopy.

Homotopic Example

Q

R

These are not homotopic



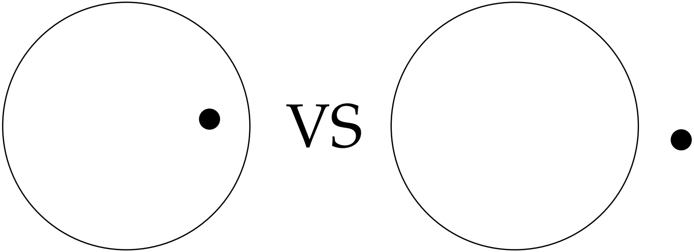
These *are* homotopic

Sort the following into homotopic classes

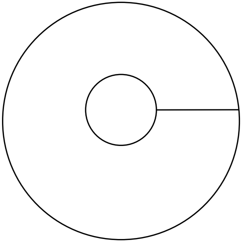
2 4 6 8

A B C D

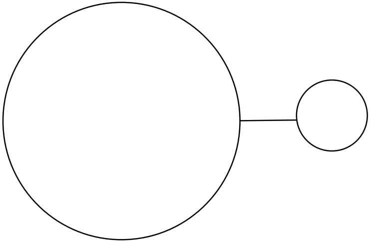
Are they homotopic?



Are they homotopic?



VS



Two-Manifolds

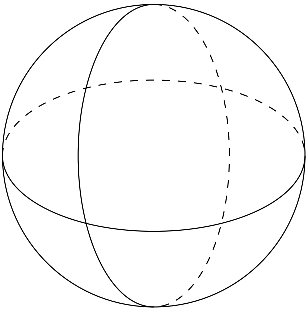
A TWO-MANIFOLD is a space that *locally* feel like the surface of the plane.

An example of a non-orientable surface

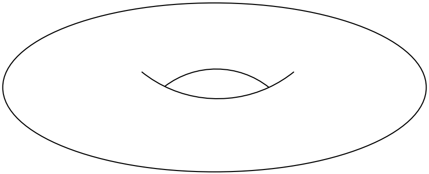
Mobius strip

Note: the Mobius strip is NOT a two-manifold.

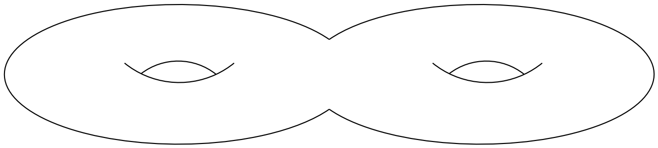
Orientable two-manifolds: Sphere



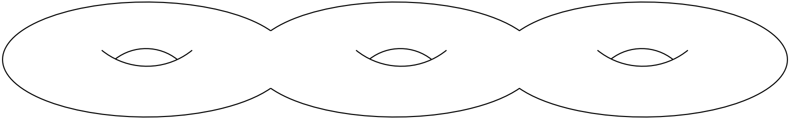
Orientable two-manifolds: Torus



Orientable two-manifolds: Connected sum of two tori



Orientable two-manifolds: Connected sum of three tori



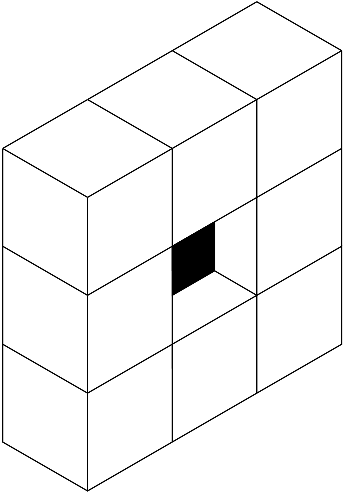
Definition: Genus; page 234

The *genus* of a two-manifold is the number of consecutive closed circular cuts we can make on the surface without disconnecting it.

Definition: Euler Characteristic; page 234

The *Euler Characteristic* of a two-manifold is $V - E + F$ where V is the number of vertices, E is the number of edges, and F is the number of polygonal faces in ANY tiling of the surface.

Tiling a Torus, Euler characteristic



V =

E =

F =

V−E+F =

Definition: Euler Characteristic

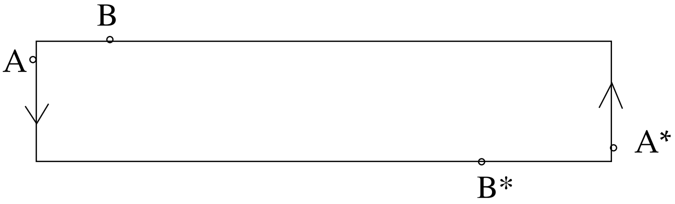
If X is a surface, denote the Euler characteristic of X by $e(X)$ and denote the genus of X by $g(X)$ then:

$$e(X) = 2 - 2g(X)$$

Some non-orientable two-manifolds

Klein Bottle

Projective Plane



Classification of two-manifolds

Every orientable two-manifold is homotopic to a sphere, a torus, or a connected sum of (any finite number of) tori.

Every non-orientable two-manifold is homotopic to a projective plane,or to a connected sum of (any finite number of) projective planes.