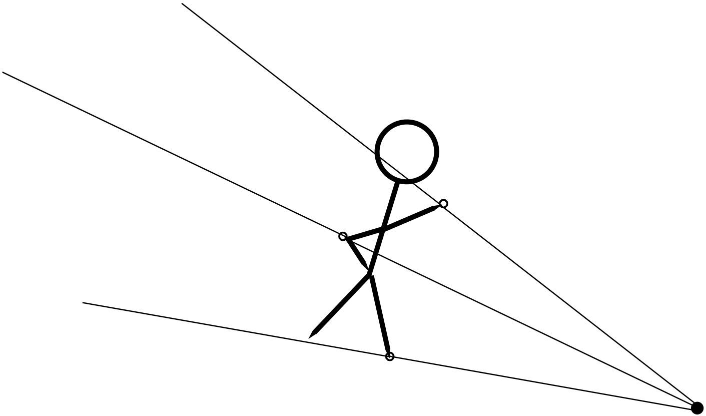
Definition, page 92

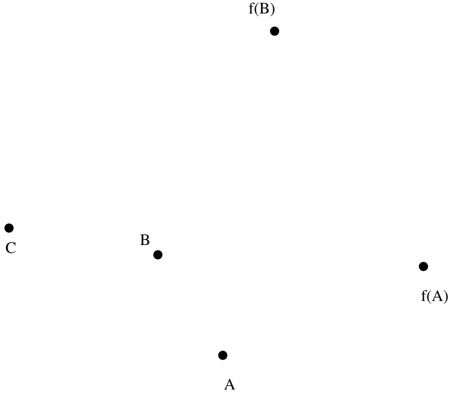
A plane transformation f is a *similarity* if there exists a positive number α such that for any two points A and B on the plane, we have $f(A)f(B) = \alpha AB$.

The number α is the stretching factor of the similarity.

Central Similarities (Dilations)



Find the center of the central similarity f, and find the image of C under f.

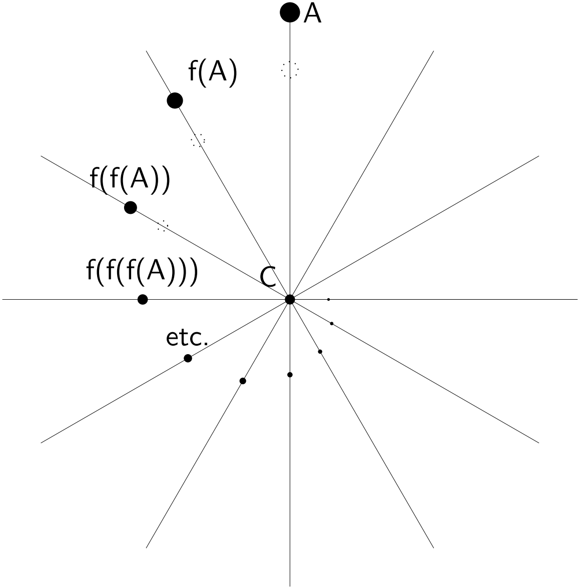


Spiral similarity

If a transformation is the composition of a rotation and a central similarity then that transformation will be a similarity. If the center is the same point for both then that composition is called a *spiral similarity*.

Spiral similarity example

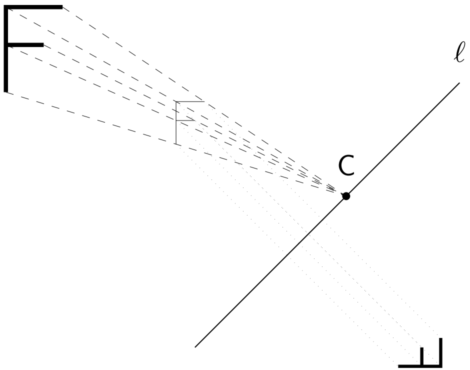
$\alpha = 0.8$, $\theta = 30^\circ$, center $= C$.



Dilative reflection

If a transformation is the composition of a a central similarity and a reflection then that transformation will be a similarity. If the center of the central similarity is on the line of reflection, then that composition is called a *dilative reflection*.

Dilative reflection example
$\alpha = 0.5$, center $= C$, line $= \ell$



Classification Theorem for Similarities

Every similarity is a symmetry, a spiral similarity, or a dilative reflection.

Squaring transformation

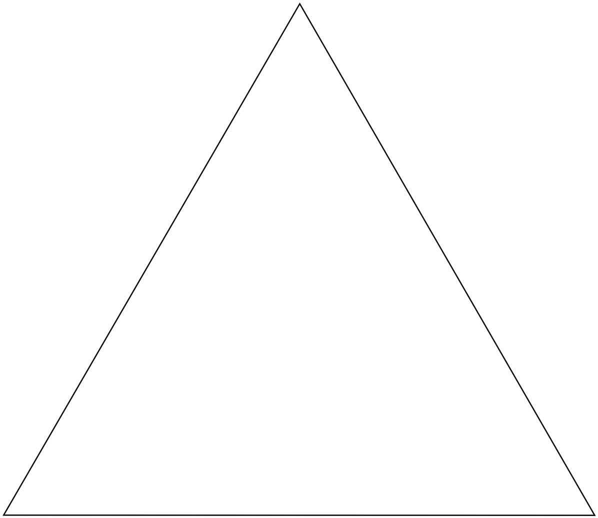
Square the distance, double the angle.

Fractals

A *fractal* is an object O possessing the property of proper self-similarity.

This means that there is a part of O, say A_1, which is similar to a proper part of O, say A_2.

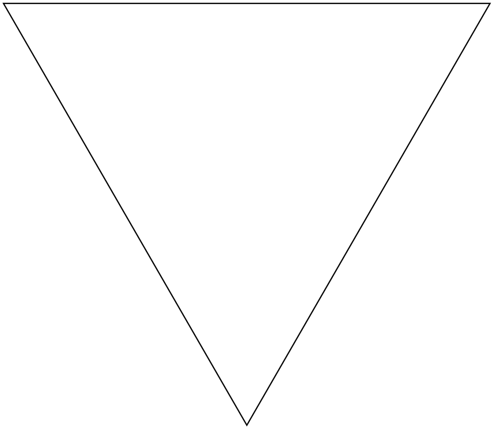
Fractal example: Sierpinski Triangle



Fractal example

Fractal example

Fractal Example



Escape Time Fractals

Given a transformation f:

The *prisoner set* is the set of points A where the set $\{A, f(A), f(f(A)), \ldots\}$ is bounded.

The *escape set* is the set of points A where the set $\{A, f(A), f(f(A)), \ldots\}$ is unbounded.

The *Julia set* is the boundary between the prisoner set and the escape set. The prisoner set is also sometimes called the *filled* Julia set.