When is a \lor -semilattice a lattice?

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ABSTRACT. In S_0 , the category of all join semilattices with 0, the tensor product has all the usual properties as in the category of abelian groups. Since join semilattices are locally finite and finite 0-semilattices are lattices, so the tensor product of two finite join semilattices is finite and a lattice. This result does not carry over to the infinite case; as shown by G. Grátzer and F. Wehrung, the tensor product of M_3 , the 5-element lattice with 3 atoms, and F_3 , the free lattice on 3 generators, is not a lattice. They then define a related construction, called the lattice tensor product, and give a very computational proof that it always yields a lattice. This paper gives an easier, more conceptual, proof using the concept of an upper bounded join homomorphism.