Skolemizing distributive semilattices

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Abstract. A (meet) semilattice $\boldsymbol{S} = \langle S; \wedge \rangle$ is *distributive* if it satisfies the following first order sentence:

$$\forall x_1, x_2, x_3\ \exists y_1, y_2\, (x_3 \geq x_1 \wedge x_2) \Rightarrow (y_1 \geq x_1) \text{ AND } (y_2 \geq x_2) \text{ AND } (y_1 \wedge y_2 = x_3). \tag{DSL}$$

The usual method of skolemizing (DSL) is to introduce two ternary operations, $d_i(x_1, x_2, x_3)$ for $i = 1, 2$, satisfying these universally quantified implications:

$$\forall x, y, z\, (z \geq x \wedge y) \Rightarrow (d_1(x, y, z) \geq x), \tag{DSI1}$$

$$\forall x, y, z\, (z \geq x \wedge y) \Rightarrow (d_2(x, y, z) \geq y), \tag{DSI2}$$

$$\forall x, y, z\, (z \geq x \wedge y) \Rightarrow (d_1(x, y, z) \wedge d_2(x, y, z) = z), \tag{DSI3}$$

We seek a class of algebras $\mathcal{SWD}$ (called *semilattices with distribution*) of the form $\langle S; \wedge, d_1, d_2 \rangle$ such that (DSI1)-(DSI3) hold (this is easy), such that $\mathcal{SWD}$ is a finitely based equational class (this is not too hard), and such that the arithmetic in $\mathcal{SWD}$ is easy (this is an entirely different matter).