Young tableaux and jeu-de-taquin promotion

Abstract

Given a partition λ of a positive integer n, jeu-de-taquin promotion is a bijection on the set of standard Young tableaux, $j : SYT(\lambda) \to SYT(\lambda)$ and column strict tableaux $\partial : CST(\lambda) \to CST(\lambda)$. The order of promotion on $SYT(\lambda)$ or $CST(\lambda)$ is the least positive integer k such that $j^k(T) = T$ for all $T \in SYT(\lambda)$ or $\partial^k(T) = T$, for all $T \in CST(\lambda)$, respectively. Given particular partitions of λ, such as $\lambda = (r^c) = (r, r, \ldots, r)$ or $\lambda = (sc_k) = (k, k-1, k-2, \ldots, 2, 1)$ the order of promotion on $SYT(\lambda)$ is known. In general the order of promotion on $SYT(\lambda)$ or $CST(\lambda)$ is not known.

A proof that the order of promotion on the set of column strict hook shapes, that is, $CST((m, 1^{n-m}))$ is $k \cdot lcm(S)$ where $S = (n-1, n-2, \cdots, n-m+1)$ will be sketched.