

1 (a) not defined $A_{2 \times 3} \neq C_{3 \times 2}$

(b) defined

$$B^T = \begin{pmatrix} 0 & 1 & -1 \\ -2 & 2 & 3 \end{pmatrix} \quad A - B^T = \begin{pmatrix} 3 & -3 & 0 \\ 4 & -3 & -3 \end{pmatrix}$$

$$(A - B^T)F = \begin{pmatrix} 4 \cdot 3 - 3 \\ 4 \cdot 4 - 3 + 9 \end{pmatrix} = \begin{pmatrix} 9 \\ 22 \end{pmatrix}$$

(c) defined $ED = (3 - 2 \quad 6 + 2 - 1 \quad -3 - 2 + 1) = (1 \quad 7 \quad -4)$

(d) ~~not defined~~ $\mathbb{R}_{3 \times 1}$ $\mathbb{R}_{1 \times 3}$

defined

$$FE = \begin{pmatrix} 12 & 8 & 4 \\ 3 & 2 & 1 \\ -9 & -6 & -3 \end{pmatrix}$$

(e) defined $F^T E^T = (4 \ 1 \ -3) \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = (12 + 2 - 3) = (11)$

(f) defined

$$C^2 = \begin{pmatrix} 4 - 1 & -2 - 3 \\ 2 + 3 & -1 + 9 \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ 5 & 8 \end{pmatrix}$$

2. (a) Not in R.E.F - 3rd row's leftmost nonzero entry is not 1.

$$R_3 \rightarrow \frac{1}{2}R_3$$

(b)

$$\xrightarrow{R_3 \rightarrow \frac{1}{2}R_3} \begin{pmatrix} 1 & 1 & 0 & 2 & 3 & 4 \\ 0 & 1 & 3 & 2 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 4 \end{pmatrix} \xrightarrow{\begin{matrix} R_2 \rightarrow R_2 - 2R_3 \\ R_1 \rightarrow R_1 - 2R_3 \end{matrix}} \begin{pmatrix} 1 & 1 & 0 & 0 & 5 & -4 \\ 0 & 1 & 3 & 0 & 3 & -9 \\ 0 & 0 & 0 & 1 & -1 & 4 \end{pmatrix} \xrightarrow{R_1 \rightarrow R_1 - R_2}$$

$$\begin{pmatrix} 1 & 0 & -3 & 0 & 2 & 5 \\ 0 & 1 & 3 & 0 & 3 & -9 \\ 0 & 0 & 0 & 1 & -1 & 4 \end{pmatrix}$$

R.R.E.F.

$$3. \left(\begin{array}{ccc|c} 1 & 1 & -2 & -2 \\ 0 & 1 & 2 & 3 \\ 1 & 2 & 0 & 1 \end{array} \right) \xrightarrow{R_3 \rightarrow R_3 - R_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{array} \right) \xrightarrow{R_3 \rightarrow R_3 - R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

z - free variable

$$y + 2z = 3$$

$$y = -2z + 3$$

$$x + y - 2z = -2$$

$$x = -y + 2z - 2 = 2z - 3 + 2z - 2 = 4z - 5$$

Ans: $x = 4z - 5$ $y = -2z + 3$

$$4. \left(\begin{array}{ccc|c} 1 & 3 & 3 & 0 \\ 2 & 7 & 2 & 6 \\ -1 & 1 & -4 & 9 \end{array} \right) \xrightarrow{\begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 + R_1 \end{array}} \left(\begin{array}{ccc|c} 1 & 3 & 3 & 0 \\ 0 & 1 & -4 & 6 \\ 0 & 4 & -1 & 9 \end{array} \right) \xrightarrow{R_3 \rightarrow R_3 - 4R_2}$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 3 & 0 \\ 0 & 1 & -4 & 6 \\ 0 & 0 & 15 & -15 \end{array} \right) \xrightarrow{R_3 \rightarrow \frac{1}{15}R_3} \left(\begin{array}{ccc|c} 1 & 3 & 3 & 0 \\ 0 & 1 & -4 & 6 \\ 0 & 0 & 1 & -1 \end{array} \right) \xrightarrow{\begin{array}{l} R_1 \rightarrow R_1 - 3R_3 \\ R_2 \rightarrow R_2 + 4R_3 \end{array}}$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array} \right) \xrightarrow{R_1 \rightarrow R_1 - 3R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array} \right)$$

Ans: $x = -3$ $y = 2$ $z = -1$

$$\begin{aligned}
 5. \quad & 2x - 3z = 2 \\
 & -x + 2y + 4t = 1 \\
 & x + 2y + z = 0
 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 2 & 0 & -3 & 0 & 2 \\ -1 & 2 & 0 & 4 & 1 \\ 1 & 2 & 1 & 0 & 0 \end{array} \right) \xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ -1 & 2 & 0 & 4 & 1 \\ 2 & 0 & -3 & 0 & 2 \end{array} \right)$$

$$\begin{array}{l} R_2 \rightarrow R_2 + R_1 \\ R_3 \rightarrow R_3 - 2R_1 \end{array} \left(\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 0 & 4 & 1 & 4 & 1 \\ 0 & -4 & -5 & 0 & 2 \end{array} \right) \xrightarrow{R_3 \rightarrow R_3 + R_2} \left(\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 0 & 4 & 1 & 4 & 1 \\ 0 & 0 & -4 & 4 & 3 \end{array} \right)$$

$$\begin{array}{l} R_2 \rightarrow \frac{1}{4}R_2 \\ R_3 \rightarrow \frac{1}{4}R_3 \end{array} \left(\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{4} & 1 & \frac{1}{4} \\ 0 & 0 & 1 & -1 & -\frac{3}{4} \end{array} \right) \begin{array}{l} R_2 \rightarrow R_2 - \frac{1}{4}R_3 \\ R_1 \rightarrow R_1 - R_3 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & \frac{3}{4} \\ 0 & 1 & 0 & \frac{5}{4} & \frac{7}{16} \\ 0 & 0 & 1 & -1 & -\frac{3}{4} \end{array} \right) \xrightarrow{R_1 \rightarrow R_1 - 2R_2} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{3}{2} & -\frac{1}{8} \\ 0 & 1 & 0 & \frac{5}{4} & \frac{7}{16} \\ 0 & 0 & 1 & -1 & -\frac{3}{4} \end{array} \right)$$

Ans: $x = \frac{3}{2}t - \frac{1}{8}$ $y = -\frac{5}{4}t + \frac{7}{16}$ $z = t - \frac{3}{4}$.