

1. $\overline{-2+3i} = -2-3i$ is also a root

$(x+2-3i)(x+2+3i) = x^2+4x+13$ divides the pol.

$$\begin{array}{r}
 x^2 - 2x + 5 \\
 x^2 + 4x + 13 \overline{) x^4 + 2x^3 + 10x^2 - 6x + 65} \\
 \underline{x^4 + 4x^3 + 13x^2} \\
 -2x^3 - 3x^2 - 6x \\
 \underline{-2x^3 - 8x^2 - 26x} \\
 5x^2 + 20x + 65 \\
 \underline{5x^2 + 20x + 65} \\
 0
 \end{array}$$

$$x^2 - 2x + 5 = 0$$

$$x = \frac{2 \pm \sqrt{4 - 4 \cdot 5}}{2} = 1 \pm \sqrt{-4} = 1 \pm 2i$$

Ans: $-2-3i, 1+2i, 1-2i$

$$2. \quad 4k = P(-2) = -8 + k^2 \cdot 4 + 8 + 1$$

$$4k^2 - 4k + 1 = 0$$

$$(2k-1)^2 = 0$$

Ans: $k = \frac{1}{2}$.

$$3. \quad \begin{cases} P(-1) = 0 \\ P(3) = 0 \end{cases}$$

$$\begin{cases} 1 - k + h + 16 - 24 = 0 \\ 81 + 27k + 9h - 48 - 24 = 0 \end{cases} \quad (\text{div. by } 3)$$

$$\begin{cases} -k + h - 7 = 0 \\ 9k + 3h + 27 - 16 - 8 = 0 \end{cases} \quad (\text{div. by } 3)$$

$$\begin{cases} -k + h - 7 = 0 & (1) \\ 3k + h + 1 = 0 & (2) \end{cases}$$

(2) - (1): ~~4~~ $4k - 8 = 0 \quad k = 2$

(1): ~~h~~ $h = k + 7 = 9$

Ans: $k = 2, h = 9$

4(a) Coefs of $P(x)$: $2, -5, -8, -6, 6, 5$ 2 sign changes
 2 or 0 positive sols.

Coefs of $P(-x)$: $-2, -5, 8, -6, -6, 5$ 3 sign changes
 3 or 1 negative sols.

(b) If x is a solution, then

$$|x| < \frac{\max\{|-5|, |-8|, |-6|, |6|, |5|\}}{|2|} + 1 = \frac{8}{2} + 1 = 5$$

$$|x| < 5$$

(c) $\pm \frac{p}{q}$ where p divides 5, so $p = 1$ or $p = 5$
 q divides 2, so $q = 1$ or $q = 2$

$$\pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}$$

By (b), we can eliminate ± 5 .

~~By (c), we can eliminate ± 5 .~~

(a) does not provide any extra info

Ans: $\pm 1, \pm \frac{1}{2}, \pm \frac{5}{2}$.

5. By rational roots, if x is an integer sol., then x divides 14, so it may be

$\pm 1, \pm 2, \pm 7, \pm 14$.

$x=1$ doesn't satisfy: $1 - 7 + 17 - 14 \neq 0$

$x=-1$ doesn't satisfy: $-1 - 7 - 17 - 14 \neq 0$

$x=2$ does: $8 - 28 + 34 - 14 = 0$

$$\begin{array}{r}
 x^2 - 5x + 7 \\
 x-2 \overline{) x^3 - 7x^2 + 17x - 14} \\
 \underline{x^3 - 2x^2} \\
 -5x^2 + 17x \\
 \underline{-5x^2 + 10x} \\
 7x - 14 \\
 \underline{7x - 14} \\
 0
 \end{array}$$

$$x^2 - 5x + 7 = 0$$

$$x = \frac{5 \pm \sqrt{25 - 28}}{2} = \frac{5 \pm i\sqrt{3}}{2}$$

Ans: $2, \frac{5 \pm i\sqrt{3}}{2}$.