

Math 1210 Tutorial #10 (Nov. 17 – Nov. 23)

1. Consider the matrix $\left(\begin{array}{ccc|c} 1 & -1 & k^2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & k \end{array} \right)$, where k is a real number. Assuming that this matrix is the augmented matrix of a non-homogeneous linear system of equations, find (the answer may depend on the value of k):

- (a) the number of variables in this system,
- (b) the rank of the augmented matrix,
- (c) the rank of the coefficient matrix,
- (d) the number of “free” variables in this system,
- (e) the number of solutions of the system.

2. Consider the non-homogeneous system of linear equations

$$x + 2y + z - w = 4, \quad 2x + y - 3z = 7, \quad w + 7t = 8.$$

- (a) Find all basic solutions of the corresponding homogeneous system (with the same coefficient matrix).
- (b) Using the results of part (a) and the fact that $(x, y, z, w, t) = (2, 0, -1, 1, 1)$ is a solution of the given non-homogeneous system, find all solutions of the system.

3. Prove that the points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) belong to the same line in \mathbb{E}^2 if and only if

$$\det \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} = 0.$$

4. Suppose that A, B, C are invertible matrices of the same size. Show that $A^T B C$ is invertible and express its inverse in terms of A^{-1}, B^{-1}, C^{-1} .

5. Suppose that A, B are $n \times n$ matrices, I is the identity matrix of the same size, and A is invertible. Show that

$$BA = A^{-1}(I + AB)A - I.$$