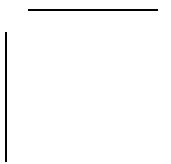
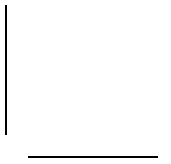




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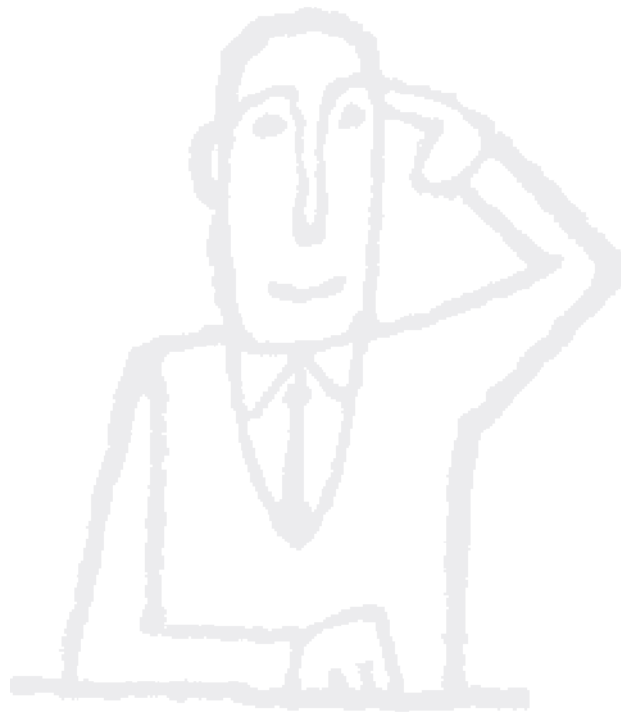


Train Your Brain

A Year's Worth of Puzzles

George Grätzer

Translated by Tom Artin



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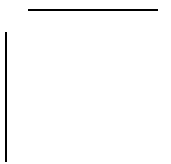
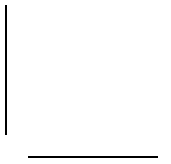
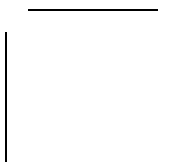
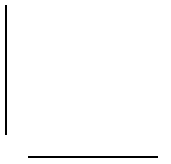


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Preface

Many of us start our day with exercise. No wonder, we've always been told about the importance of regular fitness training—Rasmussen Reports found that over 80% of Americans believe that regular exercise is important. At the same time, very few of us concern ourselves with exercising our brain, the organ that makes us human. Many of us are even proud of this. . . .

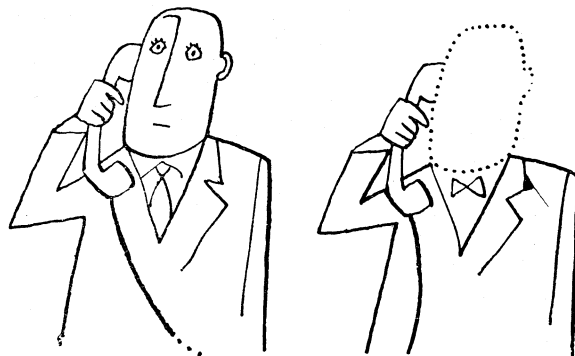
Telling riddles is one of my great passions. If I happen to hear an interesting brain teaser, I won't rest until I've passed it on to all my friends. Occasionally, though—to my great chagrin—I run into surprising resistance. If the puzzle is a bit longer, some people will break right into the middle of it and start talking about something else. "That reminds me that. . . ." If the teaser is a short one, though, and makes a telling point, others will laugh and add mechanically, "Ah, a well-rested brain thought that one up!"

Yes, fortunately, there are still people with "well-rested brains."

Unfortunately, most people don't appreciate systematic thought and the power of new ideas. In school, it is math that's most responsible for developing our logical thinking. For the majority of students, however, math is a collection of formulas to be crammed for tests. Is it really necessary to demonstrate this? Just look at how the teaching of math in school is depicted in literature.

And beyond school? After graduation, "brain training," minimal to begin with, almost ceases, and we are fully occupied with the routines of everyday life.

We really don't know what all our intellects are capable of. Even absent a Rasmussen Report, we can carry out a small statistical survey among our own friends. Let's challenge them, one at a time, with the following simple brain teaser:



“Someone called me on the phone,” explains Jack, “and when I asked with whom I was speaking, the person calling was surprised I didn’t recognize the voice, since the mother-in-law of his mother is my mother. I didn’t believe what the caller was saying, since I have no siblings.”

What is the relationship between Jack and the caller?

Most people to whom I present this puzzle ask me to repeat it, to give them a chance to pull themselves together, or maybe change the subject. Some say confidently, “It was Jack’s mother who called him, right?” And then I have to argue that this is wrong. It occurs only to a few of them to examine their answer and perhaps reach that high peak from which one doesn’t just guess, but solves the problem through logical thinking.

And really, the answer is quite simple. (It helps to draw little circles on a paper to diagram the people and then lines between the circles to describe their relationships.) The mother-in-law of the caller’s mother is Jack’s mother; in other words, the husband of the caller’s mother and Jack are children of the same mother. But since Jack has no siblings, the husband of the caller’s mother is none other than Jack himself. That is, Jack is the husband of the caller’s mother, and thus the caller is Jack’s child (or perhaps, his wife’s child).

Everybody is astonished: the caller, since the father hasn’t recognized his child’s voice, and the reader, seeing how simple the solution of such a puzzle can be.

“Really, even I could have solved that,” you might say—and you would be correct. Try out the brain teasers in this book, and you’ll see that you can solve them too.

What are the prerequisites for solving brain teasers?

First, you need to have good ones! Please browse through the book. I hope that even a picky reader will find ample material to his or her taste.

Second, you have to be committed to thinking logically. You have to think attentively and systematically through what you know, and then consider what conclusions can be drawn from it. This is not in and of itself a prescription for solving problems (no such prescription unfortunately—or thank God—exists), but rather it is a basic observation.

Third, you need an idea. Actually, not every brain teaser requires this. The problem just presented, for instance, unraveled itself, as it were, after a bit of logical reflection and without any brilliant ideas. With many brain teasers, however, you have to step a bit “outside the box,” become mental “innovators.” At first, of course, you require only modest ideas. But later, in proportion with the level of difficulty, the pleasure you feel at finding a good idea is correspondingly greater.

I have yet to find among my friends anyone who, when reading a book, is able to stick to the principles and instructions of the preface. Nevertheless, I would like to offer such advice. The reader can make use of it as he or she likes. Cheers!

As the reader will perhaps have noticed, this book is not organized into chapters, but into 52 weeks. I’ve organized the book this way so that the reader will find intellectual treats for every week of a year. The fact that each week presents only a few such treats (in the first 36 weeks, three per week; in the last 16 weeks, only two per week), counsels patience.

Take your time solving the problems. Don’t run headlong to the solution, and don’t pass the problem over lightly either, just because you think, “I’m not going to be able to solve this.” Sure you will. You’ll manage if you persist.

If you don’t solve the problems in the order in which they are presented, it can well happen that you run into unfamiliar concepts. So I recommend that you solve the puzzles in order. Also, you should always look at the solution presented in the book, even when you solve the problem independently. Often, the solution we give introduces concepts you will need later on or presents general principles that can be profitably put to use in problems that come later. Also, if necessary, consult Section 1 of the Appendix, p. 221, surveying the terminology we use.

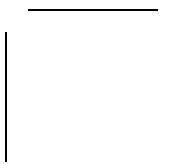
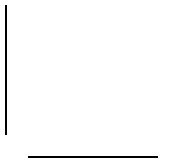
If after several attempts, you are still unsuccessful in solving a brain teaser (after repeatedly trying and reflecting on it for, let’s say, three or four days), you should first look at the *hints* in the second part of the book. There you will find ideas for solutions to a number of problems. These are not complete solutions, which are given in the third part, *Solutions*. In *Hints*, you may also find references to earlier problems—where needed. The little “Hint-Man” greeting the reader in the margin indicates that there are ideas and concepts for the solution in *Hints*.



A separate title designates the last sixteen weeks: *Black Belt*. In this part, we pose only two brain teasers per week, but if you manage two

such puzzles, you can give yourself a pat on the back. I recommend this section to all who get a kick out of experiencing beautiful ideas.

Off we go! Readers and solvers, he who seeks, finds. Look for the solution and find salvation! Have fun with it. Train Your Brain. Now it's time for our weekly brain teasers!



A Friendly Afterword to the Reader

I turn to you, now, dear reader. Just now, having solved every problem, you meant to put the book down, thinking you've finished it. I direct myself also to those curious readers who may just have thumbed through the pages to the end, without having diligently solved all the problems. Or could it be you've not even started solving them?

Let me say a few words about why, how, and from what sources this book was written.

In retrospect, be it said this book belongs to the large family of books meant to popularize mathematics. Its primary goal, as set forth in the foreword, is calisthenics for the brain. To reach this goal, though, I've chosen particular puzzles not so much for their own sake; rather, I've picked brain teasers that acquaint the reader with mathematics, and even make the experience pleasurable.

Given the book's design, the scope of mathematics it teaches is rather modest, but that is only fitting, for the reader must discover the theorems presented here for himself. In several of the brain teasers, interesting mathematical theorems are reduced to a series of elementary steps leading to the result. Think, for example, of the series of problems leading to the theory of Nim games, or of the puzzles that lead to the Eulerian condition for the traversability of graphs. A discovery like that is a terrific experience! After one's first such experience, mathematics is no longer just a collection of theorems created by some god (or by humans, but who knows how?). We realize that we too can create theorems!

Given that this math book is a work of popular science, it follows, of course, that much of its content is not new. "Nothing new under the

sun” is an adage applicable also to popular math—or at least rarely does something new appear. But those readers at whom this book is aimed presumably won’t be exhaustively versed in the world literature of popular mathematics. So, for them, it’s all the same if a brain teaser here is based on a puzzle that has appeared elsewhere or is even a familiar classic. It may well be that a classic brain teaser is all the better for having stood the test of time and assumed a corresponding aura.

My collection of brain teasers relies heavily on the beautiful book *Mathematical Recreations* by M. Kraitchik, (London, George Allen & Unwill Ltd, 1955). I’ve also borrowed a number of problems from the Hungarian *Mathematical and Physical Journal for Secondary Schools*.⁹

Finally, let me note that in putting this book together, I’ve also used brain teasers that are not puzzles in the strict sense of the word. In those cases, however, some mathematical goal has always been in the background.

Thanks for your attention, and once again, have fun!

George Grätzer

⁹The Hungarian title is *Középiskolai Matematikai és Fizikai Lapok*.

Afterword to the English Translation

I encountered the young man in Winnipeg at the Polo Park shopping center.¹⁰ He was, of course, much faster than I was, so I saw only his back as he walked past me. I thought he looked familiar. He was thin, had thick brown hair, and was in a big hurry. I caught up with him as he stopped in front of a store. He turned half way towards me and knew at once who I was. I can't say he was overjoyed to see me.

"Things haven't gone all that badly," I said, in a somewhat defensive fashion.

"Really?" he replied. "You're the son of the 'Puzzle King,'¹¹ and as a junior at university, you wrote a book which met your family's expectations: your first puzzle book. So, how many puzzle books have you added to the family tradition in the half century since then?"



"Not a single one. But I've written 20 other books. I've become a researcher in the field of mathematics. The two most important books I've written were on my two main research fields, and both have appeared in several editions. I've published more than 230 research papers. And I've become an expert in mathematical typesetting. The

¹⁰To a Hungarian reader it would be clear that I am following the story line of the *Young Man* by Frigyes Karinthy (1887–1938), a Hungarian author of pithy short stories and plays.

¹¹József Grätzer (1897–1945), the Hungarian Puzzle King, wrote in the late 1930s (among other titles) the puzzle books *Rébusz* and *Sicc*, which have brought pleasure to many generations of young readers.

books I've written on this subject have found their way into the hands of more than 40,000 mathematicians."

"Good," said the young man, "I know you've worked on a lot of different things. But why haven't you pursued puzzles?"

"I didn't just quit puzzles. When I wrote this book, I was one of the leading Hungarian chess composers¹² and I won many competitions.¹³ I quit doing that too. Thinking up puzzles, creating chess problems, and mathematical research—all three processes run partly consciously and partly subconsciously. We ponder a problem for a long time, and one morning we wake up with the solution. It's possible to do our conscious thinking in two or three different areas. It's much more difficult, though, to do the same subconsciously."

The young man didn't agree. I didn't pursue my conversation with him. Perhaps he was right, after all.

* * *

This book is the translation of the second Hungarian edition of this puzzle book. How did this second edition come about after more than 50 years?

My father's book, *Sicc*, has been published some 10 times since the war. The contract with the last publisher lapsed in 2007 and I was approached by Ádám Halmos, another Hungarian publisher, to republish *Sicc* along with *Rébusz*.

In her last year of high school, the publisher's mother, Mária Halmos, won a mathematical contest for which she was rewarded with a copy of the first edition of my puzzle book. In the course of her career, she worked with talented high school math students and she has been using my book to get them motivated.

At the meeting to discuss the publication of my father's two books, I proposed publishing a *Grätzer Trilogy*: my father's two books and my own puzzle book. With Mária present, I had an instant ally. And who can say "no" to his mother? Thus, the trilogy was published.

For the second edition, two experts have scrutinized each puzzle thoroughly: Mária Halmos, whom we've already met, and Erika Kuczmann, who for two decades pursued the teaching of mathematics to talented high school students. These two women have refined nearly every puzzle and solution and corrected many errors. It has been my great good fortune to have had these two prominent experts so fruitfully revise my book.

¹²A *chess composer* is a person who creates endgame studies or other chess problems.

¹³See <http://www.magyarsakkszerzok.com/> and <http://www.magyarsakkszerzok.com/gratzer-gy.htm>.

The book was first translated into German and from German to English. I would like to thank my publisher, Klaus Peters, and my editor, Charlotte Henderson, for their commitment to quality books. I was also assisted by Karen Kipper. Mária Halmos contributed a great deal to this project; her attention to detail is legendary.

It gives me great pleasure to know my readers can now take the product of these efforts in hand.

A handwritten signature in black ink that reads "George Grätzer". The signature is written in a cursive style with a light beige background behind the text.