

**A CORRECTION TO “SMALL REPRESENTATIONS
OF FINITE DISTRIBUTIVE LATTICES
AS CONGRUENCE LATTICES”**

GEORGE GRÄTZER, IVAN RIVAL, AND NEJIB ZAGUIA

We would like to thank Ralph Freese for pointing out that Lemma 1 of this paper (these Proceedings **123** (1995), 2619–2623) is stated and proved under the assumption “ $\Phi_1 \prec \Phi_2$ in $\text{Con } L$ ”, but it is used under the assumption “ $\Phi_1 \prec \Phi_2$ in the poset of join-irreducible congruences of L ”.

We now correctly state and prove Lemma 1:

Lemma 1. *Let L be a finite lattice, and let $v_i, u_i \in L$ satisfy $v_i \prec u_i$, for $i = 1, 2$. Let $\Phi_i = \Theta(v_i, u_i)$, for $i = 1, 2$. If $\Phi_1 \prec \Phi_2$ in the poset of join-irreducible congruences of L , then there is a three-element chain $\{e_1, h, e_2\}$ in L such that $\Phi_i = \Theta(h, e_i)$, for $i = 1, 2$, and $e_1 < h < e_2$ or $e_2 < h < e_1$.*

Proof. We assume that the reader is familiar with the basic concepts and notations of projectivity in lattices. We follow the notation on pages 129–130 of [1]. Since $v_1 \equiv u_1 (\Theta(v_2, u_2))$ and $v_1 \prec u_1$, by Theorem III.1.2 *ibid*, there is a sequence of projectivities

$$u_2/v_2 = y_1/x_1 \nearrow y_2/x_2 \searrow y_3/x_3 \nearrow \dots \searrow y_n/x_n = u_1/v_1,$$

for some natural number $n > 1$. Obviously,

$$\Phi_2 = \Theta(x_1, y_1) = \Theta(x_2, y_2) \geq \Theta(x_3, y_3) \geq \dots \geq \Theta(x_n, y_n) = \Phi_1.$$

Since $\Phi_2 > \Phi_1$, there is a smallest i satisfying $\Theta(x_i, y_i) < \Phi_2$; obviously, $3 \leq i$ and $i \leq n$.

Let i be odd, and let $z = x_{i-1} \vee y_i$. Then $\Theta(x_{i-1}, z) = \Theta(x_i, y_i) < \Phi_2$, but $\Theta(x_{i-1}, y_{i-1}) = \Phi_2$, so $\Theta(z, y_{i-1}) = \Phi_2$. Since $u_1 \equiv v_1 (\Theta(x_{i-1}, z))$ and $u_1 \prec v_1$, there are $u, v \in [x_{i-1}, z]$, $v \prec u$, such that $\Theta(u_1, v_1) \leq \Theta(u, v)$. So $\Phi_1 = \Theta(u_1, v_1) \leq \Theta(u, v) < \Phi_2$, and $\Theta(u, v)$ is a join-irreducible congruence, hence by the assumption on Φ_1 and Φ_2 , it follows that $\Phi_1 = \Theta(u, v)$. Hence we can choose $e_2 = x_{i-1}$, $h = z$, and $e_1 = y_{i-1}$.

If i is even, then we proceed dually. □

REFERENCES

- [1] G. Grätzer, *General Lattice Theory*, Pure and Applied Mathematics Series, Academic Press, New York, 1978; Mathematische Reihe, Band 52, Birkhäuser Verlag, Basel; Akademie Verlag, Berlin.

Date: Dec. 1, 1995.

1991 *Mathematics Subject Classification.* Primary 06B10; Secondary 06D05.

Key words and phrases. Congruence lattice, finite lattice, distributive lattice.

The research of all three authors was supported by the NSERC of Canada.

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF MANITOBA, WINNIPEG, MAN. R3T 2N2,
CANADA

E-mail address: `George.Gratzer@umanitoba.ca`

DEPARTMENT OF COMPUTER SCIENCE, UNIVERSITY OF OTTAWA, OTTAWA ONT. K1N 6N5,
CANADA

E-mail address: `rival@csi.uottawa.ca`

E-mail address: `zaguia@csi.uottawa.ca`